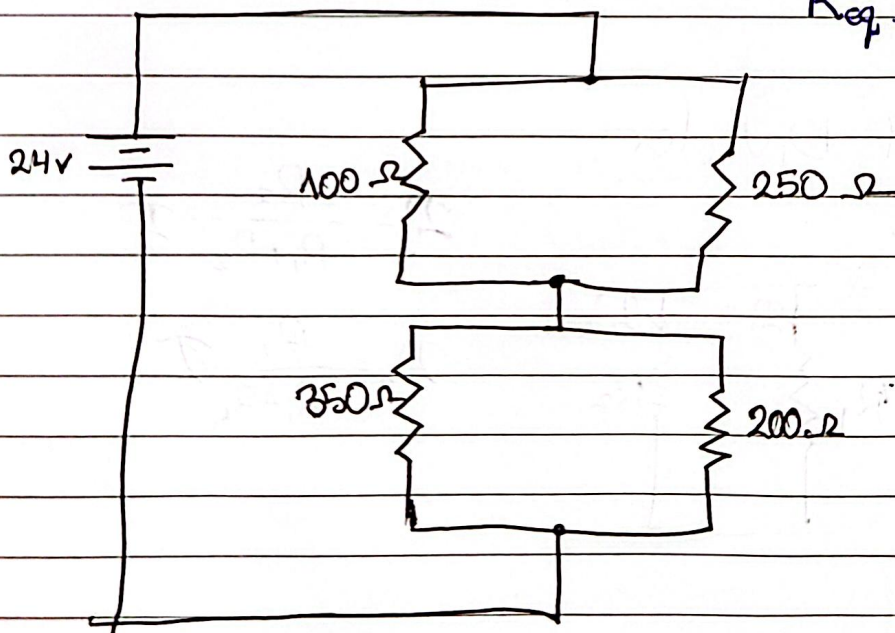


Electrical Nodes, Branches & Loops 1-Feb-2026

- # of Branches are the number of elements
- # of Nodes is the number of connection between two or ~~less~~ more branches
- # of Loops is the number of closed path formed by starting at a node, passing through a set of nodes, and returning to the starting node without passing through the same nodes

$$b = l + n - 1$$

branches loops nodes



$$R_{eq1} = \frac{1}{\frac{1}{250} + \frac{1}{100}} = 71.42 \Omega$$

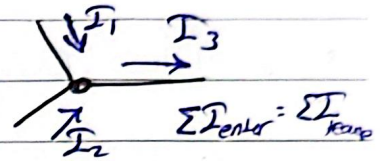
$$R_{eq2} = \frac{1}{\frac{1}{200} + \frac{1}{350}} = 127.27 \Omega$$

$$R_{eq} = R_{eq1} + R_{eq2} = 71.42 + 127.27 = 198.69 \Omega$$

$$R_{eq} = 198.69 \Omega$$

Kirchoff's Current Law (KCL)

All current in = All current out



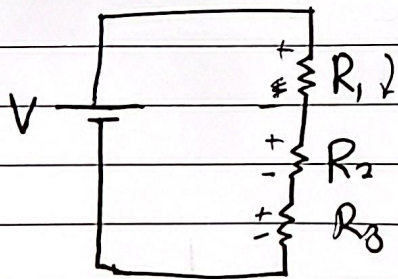
Kirchoff's Voltage Law (KVL)

All voltage in = All voltage out

$\Sigma V = 0$ for a closed loop.

~~The~~ components (resistors) ~~may~~ ^{will} have a true polarity (design) on the terminal with current going in, and a -ve where current leaving.

Voltage Divider

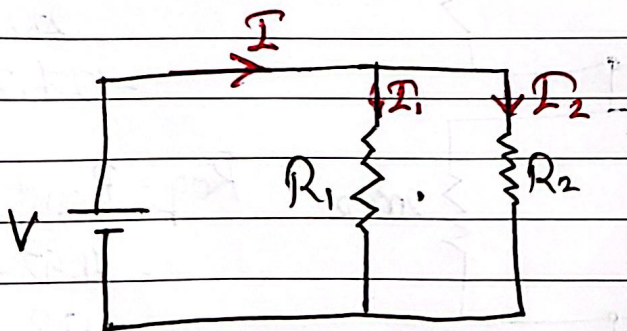


$$V_1 = \left(\frac{R_1}{R_1 + R_2 + R_3} \right) V$$

$$V_2 = \left(\frac{R_2}{R_1 + R_2 + R_3} \right) V$$

$$V_3 = \left(\frac{R_3}{R_1 + R_2 + R_3} \right) V$$

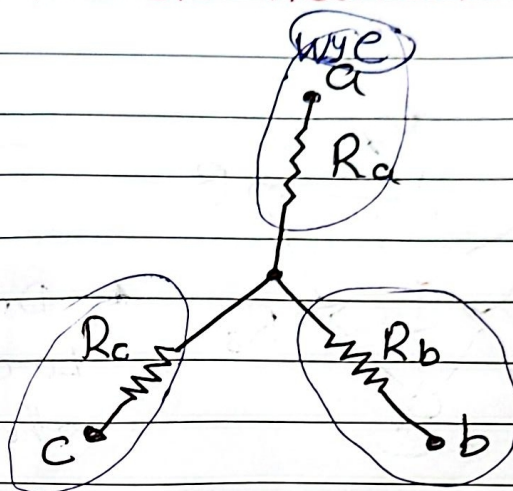
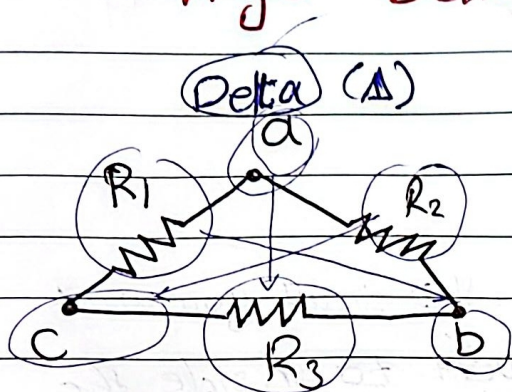
Current Divider



$$I_1 = \frac{R_2}{R_1 + R_2} I$$

$$I_2 = \frac{R_1}{R_1 + R_2} I$$

Wye - Delta Transformation



$$R_1 = \frac{R_a R_b + R_b R_c + R_a R_c}{R_b}$$

$$R_a = \frac{R_1 \cdot R_2}{R_1 + R_2 + R_3}$$

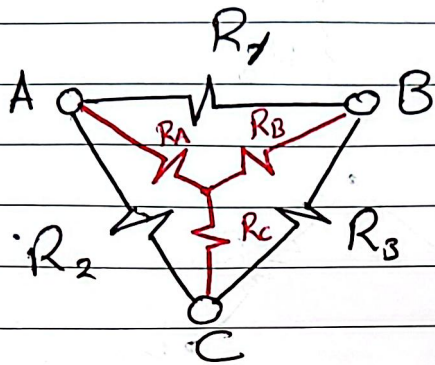
$$R_2 = \frac{R_a R_b + R_b R_c + R_a R_c}{R_c}$$

$$R_b = \frac{R_2 \cdot R_3}{R_1 + R_2 + R_3}$$

$$R_3 = \frac{R_a R_b + R_b R_c + R_a R_c}{R_a}$$

$$R_c = \frac{R_1 \cdot R_3}{R_1 + R_2 + R_3}$$

Best way of converting $\Delta \rightarrow Y$ and $Y \rightarrow \Delta$



← Draw the circuit you want to convert to inside the other one (in the same diagram)

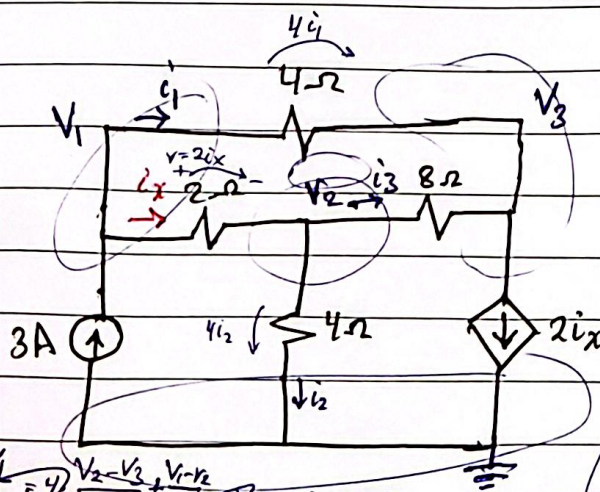
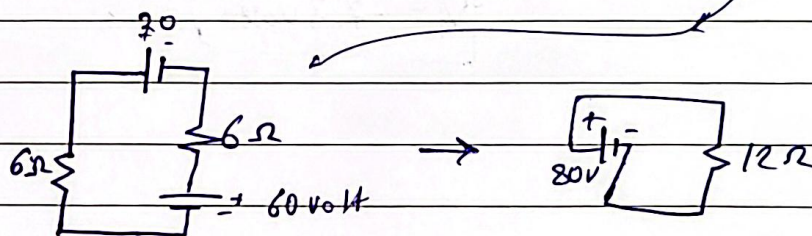
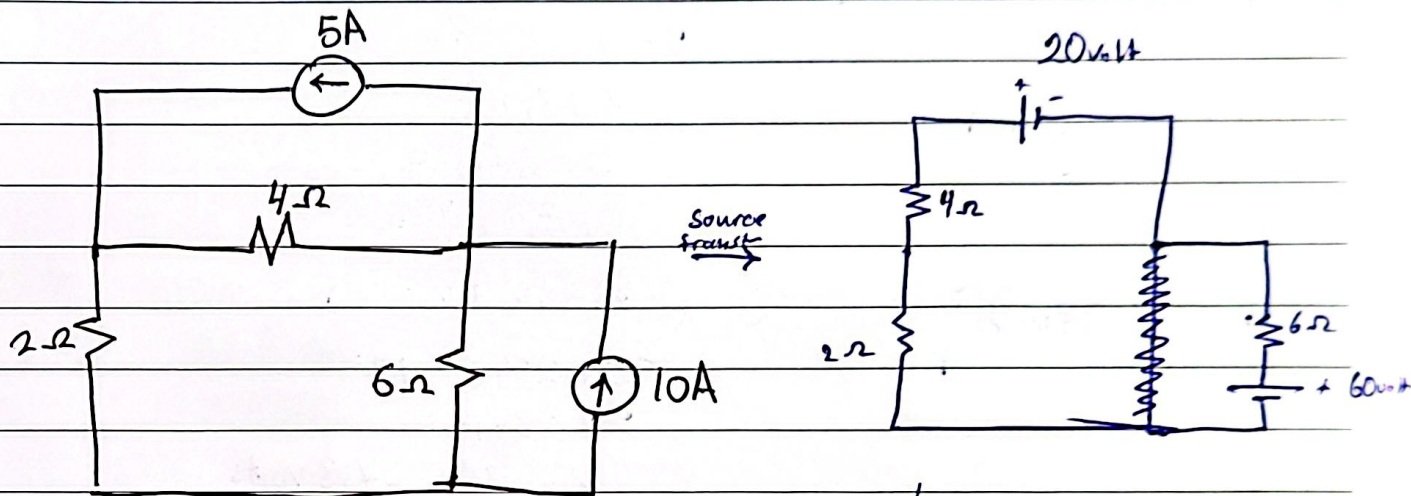
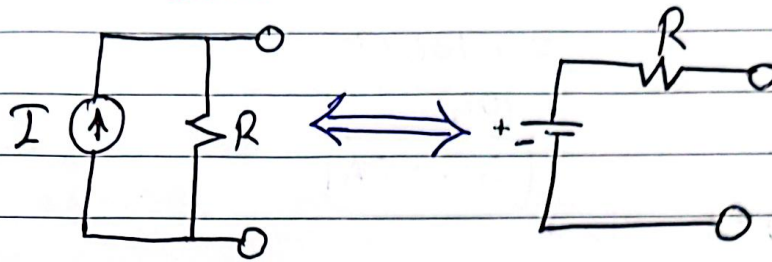
What I noticed Wye-Delta transformations are about is, when we have a certain configuration (Y or Δ), and it is connected in a certain way (certain nodes connections) then trying to spot a series or parallel config is impossible. We then convert it to other type to have a series or parallel connection.

I noticed that the Y configuration ~~has~~ resistors have more resistance (bigger in number) than the Δ configuration ~~has~~ resistors. That is because the Y configuration is more like a ~~the~~ "series", and the Δ is more like a "parallel".

Super Node and source transformation

22 Feb 2026

Source transformation



nodes: 4
branches: 6

$$i_x + i_1 = 3 \Rightarrow \frac{V_2 - V_1}{2} + \frac{V_3 - V_1}{4} = 3$$

$$2i_x + i_2 = 3$$

$$4i_1 + 8(-i_3) + 2(-i_x) = 0$$

$$4i_1 - 8i_3 - 2i_x = 0$$

$$4i_1 = 8i_3 + 2i_x$$

$$V_2 - V_1 + \frac{V_3 - V_1}{4} = 3$$

$$V_2 - V_1 + \frac{V_3}{4} = 3$$

$$4V_2 - 4V_1 + V_3 = 12$$

$$5V_2 = 3 + 4V_1$$

$$2V_2 - 2V_1 + V_3 - V_1 = 3$$

$$2V_2 + V_3 - 3V_1 = 3$$

Q. $\frac{V_2 - V_1}{2} = 4$
 $\frac{V_2 - V_1}{2} = \frac{V_2 - V_3}{8} + \frac{V_1 - V_2}{2}$
 $\frac{V_2 - V_1}{2} = \frac{V_2 - V_3}{8} + \frac{V_1 - V_2}{2}$
 $2V_2 - 2V_1 = \frac{V_2 - V_3}{4} + V_1 - V_2$
 $8V_2 - 8V_1 = V_2 - V_3 + 4V_1 - 4V_2$
 $2V_2 - 2V_1 = 0$
 $2V_2 = 2V_1$

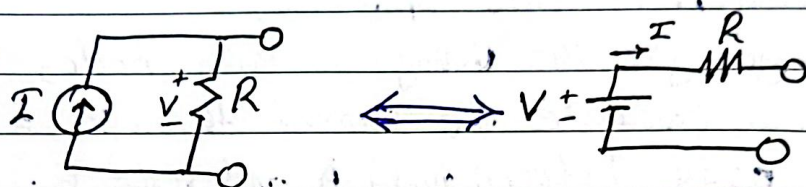
Q. $\frac{V_3 - V_1}{4} = \frac{V_2 - V_1}{2} + \frac{V_2 - V_3}{8}$
 $V_3 - V_1 = V_2 - V_1 + \frac{V_2 - V_3}{2}$
 $V_3 - V_1 = V_2 - V_1 + \frac{V_2 - V_3}{2}$
 $V_3 - V_1 = V_2 - V_1 + \frac{V_2 - V_3}{2}$

Chapter 3 - Methods of Analysis

25/ Feb/ 2023

We have Source Transformation, Nodal Analysis, and Mesh Analysis.

Source transformation



Nodal Analysis

We have to first put clear steps to follow when solving using Nodal Analysis (or mesh analysis)

Steps:

① Count and Identify the Nodes

- Circle or highlight the nodes

② Pick a ~~xxxx~~ Reference Node (Ground)

- We need a starting point for our measurement which is the ground or the reference node.
- Set the ground node to 0V
- Usually, the node connected to the most components is the best choice for ground.

③ Label the unknown Node Voltages

- Assign a variable like V_1, V_2, V_3 , etc. to every other node. (Don't add to the ground)

④ Write KCL Equations

- For every node, where voltage is unknown, write a KCL equation for it ($i_1 = i_2 + i_3 + \dots + i_n$, as ex.)

$$I = \frac{V_{\text{start}} - V_{\text{end}}}{R}$$

$$\frac{V_1 - V_x}{R_a} + \frac{V_1 - V_y}{R_b} + \dots = 0$$

Rearrange the equation so that it is in terms of the voltage nodes (V_1, V_2, V_3 , etc.)

⑤ Solve the system of Equations

- Once you write KCL equations for all nodes, you will have a system of linear equations.
- If you have 3 unknown nodes, you will have three equations. If 2 unknown nodes, you will have two equations

⑥ Find Branch Currents (Optional)

- Once you get the Voltage of each node, then you can calculate the current through a resistor R ~~among~~ between node A and node B:

$$I_{AB} = \frac{V_A - V_B}{R}$$

Mesh Analysis

steps:

① Identify the meshes.

- A mesh is a loop that does not have any other loop inside it.
- You can look at your circuit ~~di~~ diagram like it is a window frame. Identify each window pane in circuit

② Assign Mesh Currents

- For every mesh you identified, draw a circular arrow to represent the current flowing around that loop
- Give each one a name (like I_1 , I_2 , I_3 , etc.)

Tip: Always draw your arrows in the clockwise direction to prevent +ve/-ve sign mistakes.

③ Write KVL equations

- For each mesh identified, ~~write an equation~~ based on KVL for that mesh.
- As you walk around the loop in the direction of your arrow:

→ Resistors: The voltage drop is always current $\times$ Resistance ($I \times R_a$)

→ shared Resistors: If a resistor is shared between Mesh 1 and Mesh 2, then the current through it is the difference between them $(I_1 - I_2) R_a$

→ Voltage sources

if you hit the minusside of the voltage source write it as negative (e.g., $-10V$).

And if you hit +ve side write with +ve (e.g., $+10V$)

final equation for a mesh looks like this:

$$\sum V_{\text{sources}} + \sum (I \times R) = 0$$

④ Deal with current sources

- If you see a current source on a branch shared by two meshes:

→ you cannot write a standard KVL equation for those meshes because of the unknown voltage across current source

→ Combine the two meshes into one big supermesh by temporarily ignoring the branch of the current source

→ Add a constraint equation relating current in both meshes with current source ($I_{\text{source}} = I_2 - I_1$)

⑤ solve the system of Equations

• If you have 2 meshes, you will have 2 equations, 3 meshes $\rightarrow$ 3 equations

⑥ Find specific branch currents (optional)

Chapter 4

Superposition Theorem

- Helpful in determining the current across a branch or voltage across an element when the circuit contains multiple voltage and current sources.
- An advantage is we can simply add solutions obtained from analysing the network with one source activated at a time.
- In this case, the current through or voltage across an element in a bilateral network equals to the ~~voltage~~ algebraic sum of the currents or voltages produced independently by each source.

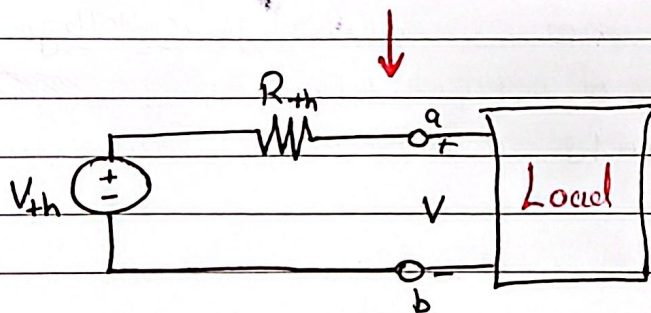
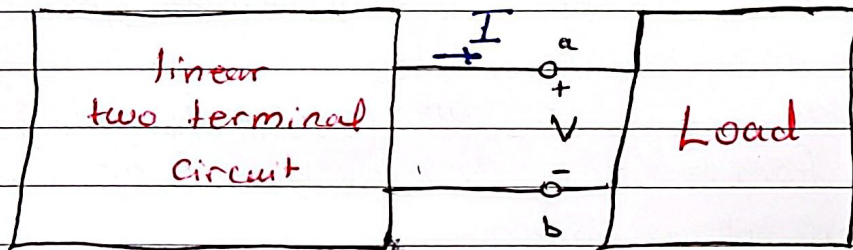
Steps:

- ① Replace Voltage sources with short circuit
 - ② Replace current sources with open circuit
- ↳ by doing so: the internal resistance of voltage source is preserved in the network, as well as the current source.
- ③ All dependent sources must be left intact as they are controlled by other circuit variables.

Thevenin's Theorem

8/3/2026

A linear two terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{th} and in series with a resistor R_{th} .



- V_{th} is the open-circuit voltage at the terminals
- R_{th} is the input or equivalent resistance at the terminals when the independent sources are turned off

→ To get V_{th} just get voltage difference between the two nodes (a and b).

→ To get R_{eq} , remove all independent sources and just simplify the resistance tree until you get the equivalent resistance. R_{th}

Thevenin's Theorem with dependent Sources

- Either connect a voltage source, eg, 1V, and find the current across that voltage source.

Then with 1V and I you have $R_{th} = \frac{V}{I} = \frac{1}{I}$

- OR put a current source and get the voltage difference across its terminal

Norton's Theorem

Very similar to Thevenin's Theorem. It is finding an equivalent to a circuit using a current source and a parallel resistor.

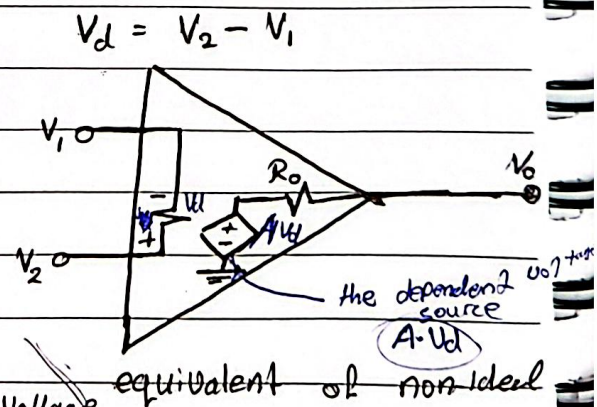
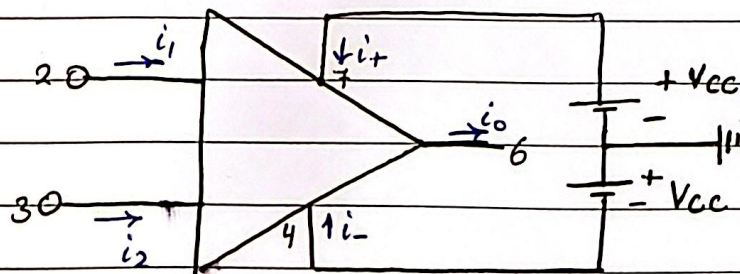
Operational Amplifier

- typically called an "op amp" for short
~~it is a voltage~~
- An electronic component that behaves like a voltage controlled voltage source
- Performs mathematical operations (Addition, Subtraction, integration, differentiation, multiplication).
- Consists of a complex arrangement of resistors, capacitors, and transistors and diodes.

There are 5 terminals found in all op-amps:

- Inverting input (often marked with -)
- Non-inverting input (often marked with +)
- Output
- Positive power supply (V_{cc})
- negative power supply ($-V_{cc}$)

As an active element in the circuit, the op-amp requires a power source. And often in circuit diagrams, the power supply terminals are obscured.



"A" is called the open-loop ~~voltage gain~~ ^{equivalent of non-ideal} op amp gain because it is the gain of the op-amp without any external feedback from output to input.

$$V_o = A \cdot V_d = A (V_2 - V_1)$$

Characteristics of ideal op amp:

Infinite Input Resistance: Meaning that there is basically no current flowing in the input terminals, which makes $V_d \rightarrow 0$

Infinite open-loop gain: The gain A is infinite large. In real devices, it ranges between 100,000 to 100,000,000.

Notice, dependent source now outputs: $A \cdot V_d = \infty \cdot 0 = \text{any possible number}$

Zero output resistance: meaning: op amp is independent of the load it is driving (supplying)

Feedback

- A "negative feedback" is achieved when the output is fed back to the inverting terminal of the op amp.
- Depending on ~~used~~ what elements this signal passes through (the feedback), the gain and behavior of the op amp changes.
- Positive feedback would lead to oscillations.
- When there is a feedback path from output to input, the ratio of the output voltage to the input voltage is called the closed-loop gain.

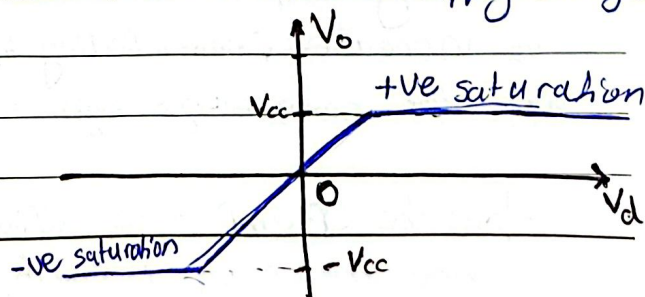
Voltage saturation

- As an ideal voltage source, the output would be unlimited voltage.
 - In reality, output voltage cannot exceed supply voltages.
 - When an output should exceed the possible voltage range, the output remains at either the maximum or minimum supply voltage.
- This is called saturation

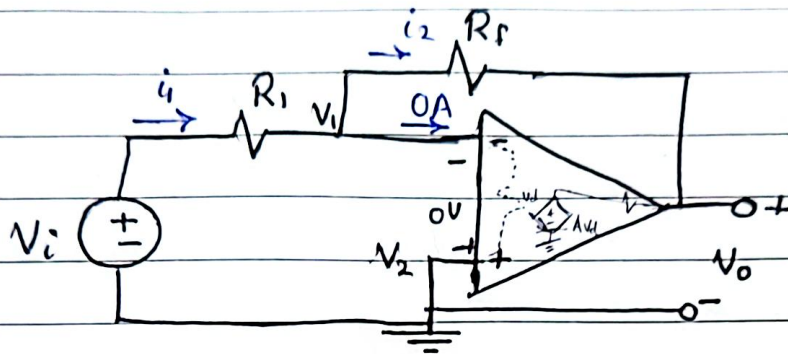
Positive saturation: $V_o = V_{cc}$

Linear region: $-V_{cc} \leq V_o = A V_d \leq V_{cc}$

Negative saturation: $V_o = -V_{cc}$



Inverting Amplifier



applying KCL at node V_1

$$i_1 = i_2 + 0$$

$$i_1 = i_2$$

$$\frac{V_i - V_1}{R_1} = \frac{V_1 - V_o}{R_f}$$

and with negative feedback established, we know that

$$V_1 = V_2 = 0$$

$$\text{So, } V_1 = 0$$

$$\frac{V_i}{R_1} = \frac{-V_o}{R_f}$$

From this equation we can see that the gain is the ratio between the feedback resistor R_f and R_1 .

$$V_o = -\frac{R_f}{R_1} V_i \rightarrow V_o = -A V_i$$

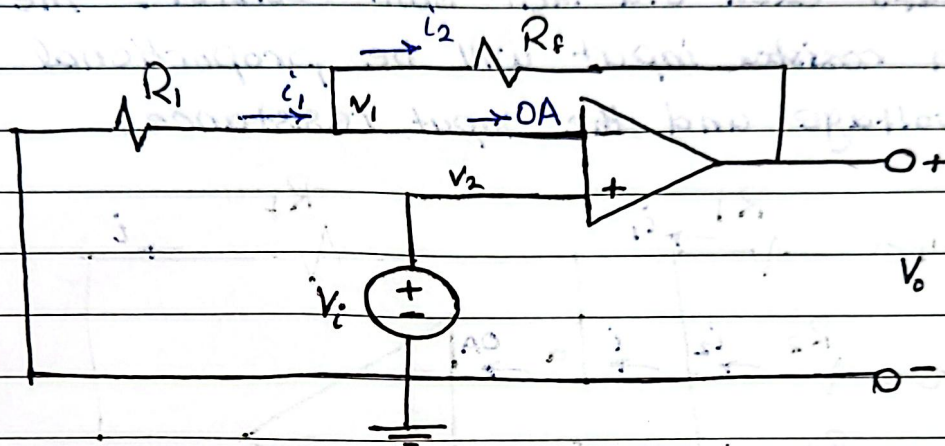
And, the polarity of the output is reverse to that of the input, thus the name "inverting" amplifier.

So, when V_1 increases (inverting input voltage), the voltage of the output increases in the opposite sign, which acts as a sucking component that helps maintain V_1 at 0 volts. And when V_2 increases (non-inverting input voltage), the output of the amp has the same voltage sign as V_2 change.

How the op amp acts differently between V_1 and V_2 goes down to how the op amp is configured from the inside with transistors etc.

Non-Inverting Amplifier

Very similar to the Inverting amplifier, except that the ground and input are switched.



$$i_1 = i_2 \Rightarrow \frac{0 - V_1}{R_1} = \frac{V_1 - V_o}{R_f} \Rightarrow \frac{-V_1}{R_1} = \frac{V_1 - V_o}{R_f}$$

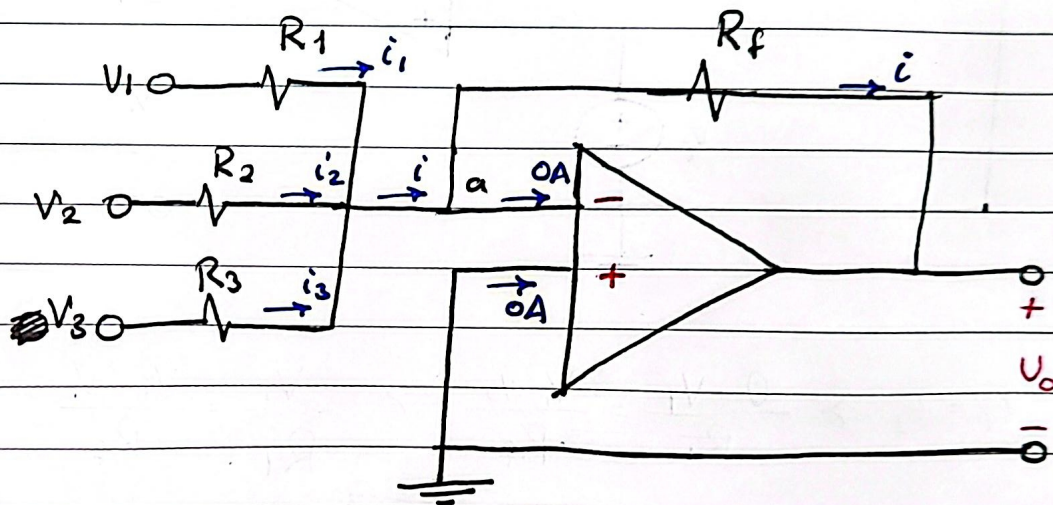
$$\Rightarrow \boxed{V_o = \left(1 + \frac{R_f}{R_1}\right) V_i}$$

$\therefore A = 1 + \frac{R_f}{R_1}$ Gain here is +ve

Summing Amplifier

20/3/2026

- Aside from Amplification, we can use an op-amp to do Addition easily. In Addition we mean outputting a voltage ~~or current~~ that is the sum of some input voltages.
- If we take an inverting amplifier and combine several inputs each via their own resistor: the current from each resistor input will be proportional to the applied voltage and the input resistance.



$$i = i_1 + i_2 + i_3$$

$$\frac{V_a - V_o}{R_f} = \frac{V_1 - V_a}{R_1} + \frac{V_2 - V_a}{R_2} + \frac{V_3 - V_a}{R_3}$$

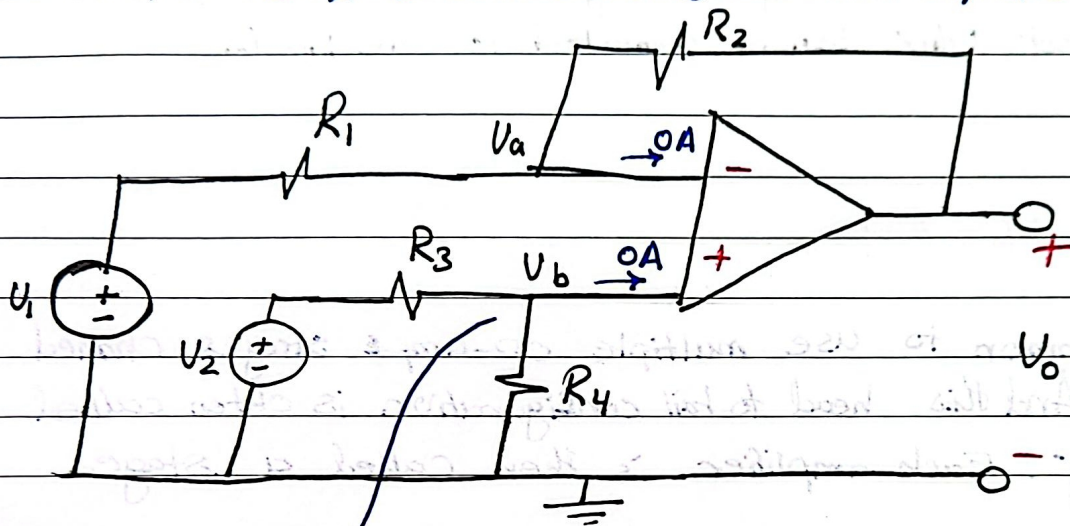
after we simplify

$V_o = - \left[\underbrace{\frac{R_f}{R_1} V_1}_{\text{scalar multiple of } V_1} + \underbrace{\frac{R_f}{R_2} V_2}_{\text{scalar multiple of } V_2} + \frac{R_f}{R_3} V_3 \right]$	and if $R_f = R_1 = R_2 = R_3$ then $V_o = -(V_1 + V_2 + V_3)$ which is an addition
--	---

So, the output is a weighted sum of the inputs

Difference Amplifier

Subtraction comes naturally to op amps since its output is ^{proportional to} difference of its inputs



Since OA is flowing through the non-inverting input, the voltage divider will ~~to~~ give the same exact voltage in V_b when we load the voltage divider same as the unloaded. And all that is thanks to infinite input resistance of op-amp. Because if the non-inverting input was taking ~~in~~ current from the loaded voltage divider, V_b value wouldn't be same as its unloaded value. ~~we~~ can also remove R_4 completely and adjust V_2 so that how much we want for it to be ~~is~~ and V_b would equal V_2 , since input resistance is infinity. ~~R3~~ will hardly take any voltage.

After applying KCL at node a and b, we get: (and knowing $V_a = V_b$)

$$V_o = \frac{R_2(1 + \frac{R_1}{R_2})}{R_1(1 + \frac{R_3}{R_4})} V_2 - \frac{R_2}{R_1} V_1$$

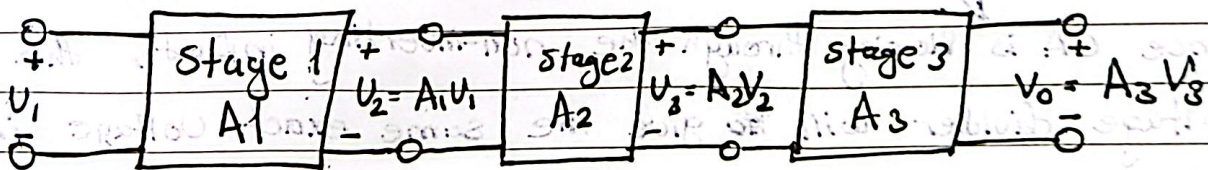
if $R_2 = R_1$ and $R_3 = R_4 \rightarrow V_o = V_2 - V_1$

Instrumentation Amplifier

This is a powerful circuit that does subtraction and enables the circuit to "ignore" any noise in the inputs that could affect other devices connected to the inputs.

Cascaded op-Amps

It is common to use multiple op-amps stages chained together. And this head to tail configuration is often called "cascading". Each amplifier is then called a stage.



Due to the ideal op-amp's input and output impedances, stages can be chained together without impacting the performance of any one stage.

One reason to cascade amplifiers is to increase the overall gain.

The gain of a cascade of Amplifiers:

$$A = A_1 A_2 A_3$$

For example, if two stages have a gain of 100 each then they combine together to 10,000.

Chapter 6: Capacitors and Inductors

Capacitor

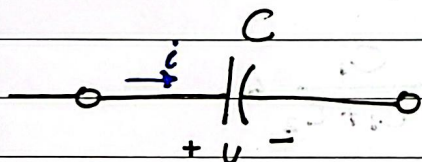
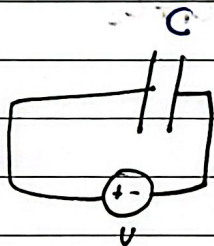
$$q = C V$$

$$C = \frac{\epsilon A}{d}$$

C: capacitance

ϵ : permittivity of the dielectric material

V: Voltage difference between the two plates



Current-Voltage relationship of capacitor

$$i = C \frac{dV}{dt} \quad \leftarrow \text{this comes from } Q = CV \text{ and } i = \frac{dQ}{dt}$$

if i is flowing into the +ve terminal of C:

• Charging: i is +ve

$$V = \frac{1}{C} \int_{t_0}^t i dt + V(t_0)$$

• Discharging: i is -ve

Energy stored in a capacitor

$$W = \frac{1}{2} C V^2$$

it came from

$$P = v i \text{ and } p = v \cdot C \frac{dV}{dt}$$

energy stored is integral of power over time

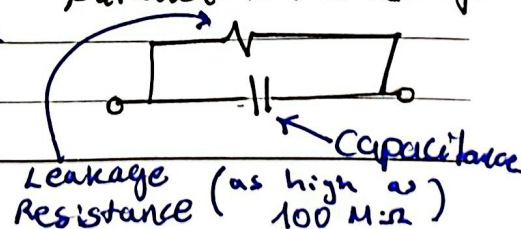
$$E = \int_{-\infty}^t P dt = \int_{-\infty}^t v \cdot C \frac{dV}{dt} dt = \int_0^V v \cdot C dV = \frac{1}{2} C V^2$$

$$w = \frac{1}{2} C V^2$$

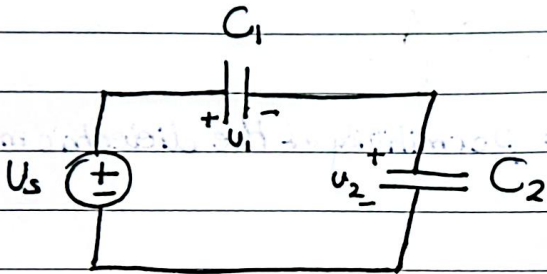
as time approaches $-\infty$, V is considered to be zero

the ideal capacitor does not dissipate energy. It takes power from the circuit when storing energy and gives previously stored energy when delivering power.

A real capacitor has a parallel-model leakage.



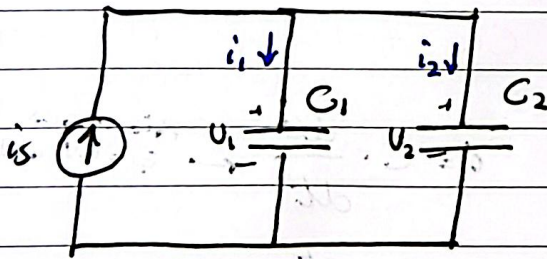
Voltage divider with capacitors



$$U_1 = \frac{C_2}{C_1 + C_2} U_s$$

$$U_2 = \frac{C_1}{C_1 + C_2} U_s$$

Current divider with capacitors



$$i_1 = \frac{C_1}{C_1 + C_2} i_s$$

$$i_2 = \frac{C_2}{C_1 + C_2} i_s$$

Inductance

$$U = L \cdot \frac{di}{dt}$$

inductance

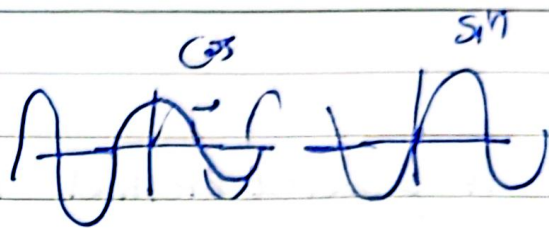
$$L = \frac{N^2 \mu A}{l}$$

Inductance is the property whereby an inductor exhibits opposition to the change of current flowing through it.

Parallel inductors

$$L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$$

Chapter 9 - Phasors

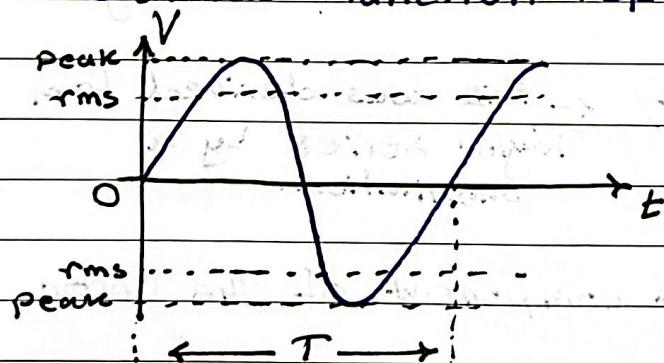


$$V_m \cos(\omega t + \phi) = V_m \angle \phi$$

$$V_m \sin(\omega t + \phi) = V_m \angle \phi - 90$$

Sinusoids

- A sinusoidal sourcing function (AC power source) produces both a transient and a steady state response. (That happens when you first open an AC source).
- When the transient has died out, we say the circuit is in sinusoidal steady state.
- A sinusoidal voltage may be represented as $v(t) = V_m \sin \omega t$
- The sinusoid function repeats itself every T seconds.

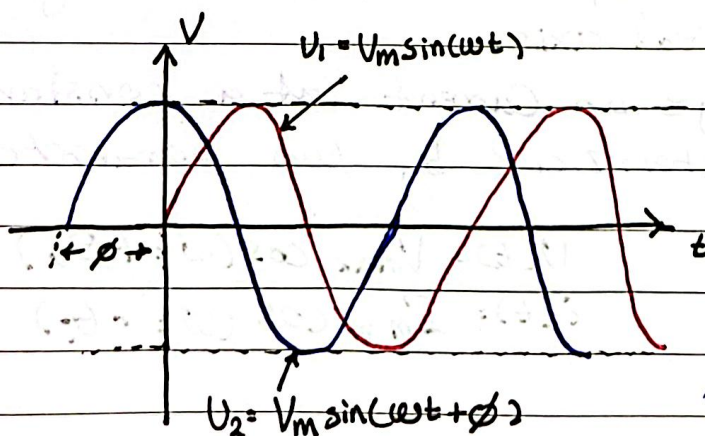


$$T = \frac{2\pi}{\omega} \quad T = \frac{2\pi}{2\pi f} = \frac{1}{f}$$

$$\omega = 2\pi f \quad f = \frac{1}{T}$$

↑
in rad/s

- We need to account for relative timing between two waves



the difference in timing between both waves is called phase shift ϕ if two sinusoids are in phase this means ϕ between them is zero

Complex Numbers

- A powerful method for representing ~~phasor~~ sinusoids is the phasor.
- The phasor works through complex numbers

complex number $Z = \underbrace{x}_{\text{real part (component)}} + j \underbrace{y}_{\text{imaginary component}}$, where $j = \sqrt{-1}$

the complex number could also be written in polar and exponential forms:

$$Z = x + jy = r \angle \phi = r e^{j\phi}$$

The exponential form depends on Euler's identity:

$$e^{\pm j\phi} = \cos \phi \pm j \sin \phi \leftarrow \text{this was derived from Taylor Series by a substitution.}$$

Phasors

Note: The sinusoid is the real component of the vector in the complex plane.

Note: The length of the vector is the amplitude of the sinusoid.

Note: The vector, V , in the polar form, is at an angle ϕ from the +ve real axis.

A sinusoidal voltage or current at a constant frequency is characterized by two parameters:

① A maximum value

② A phase angle

$$V(t) = V_{\max} \cos(\omega t + \theta_v)$$

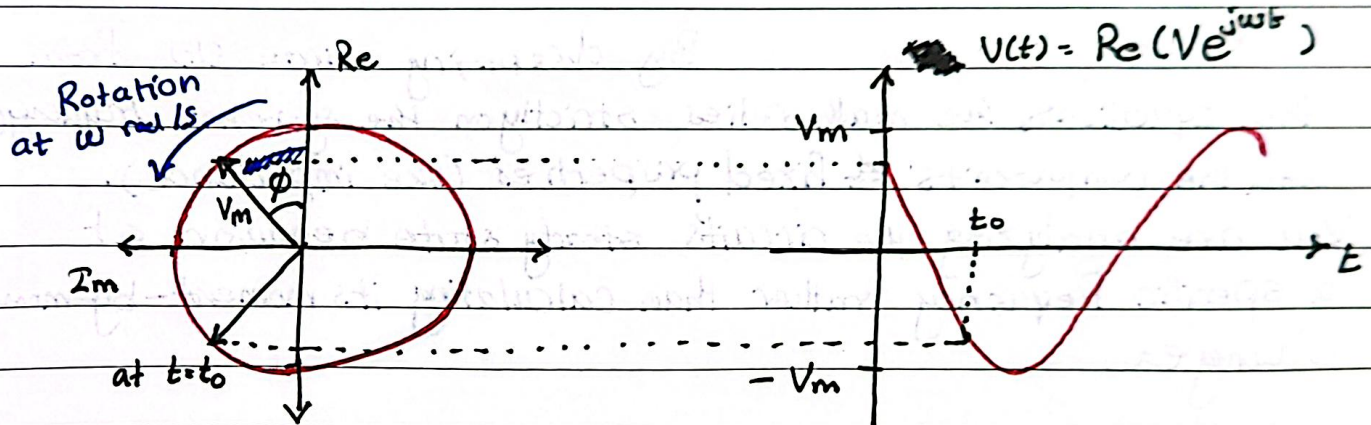
$$i(t) = I_{\max} \cos(\omega t + \theta_i)$$

- Phasors are typically represented at $t=0$
- So, the transformation from time domain to Phasor domain is:

$$V(t) = V_m \cos(\omega t + \phi) \leftrightarrow V = V_m \angle \phi$$

Time-domain
representation

Phasor - domain
representation



Time domain representation	Phasor domain representation
$V_m \cos(\omega t + \phi)$	$V_m \angle \phi$
$V_m \sin(\omega t + \phi)$	$V_m \angle \phi - 90$
$I_m \cos(\omega t + \theta)$	$I_m \angle \theta$
$I_m \sin(\omega t + \theta)$	$I_m \angle \theta - 90$

Note: when translating from Time domain to phasor domain, keep in mind that cosine function's phase is the base line.

The symbol " $\angle$ " represents the starting angle that exactly appears in the cosine function (at $t=0$).

So, " $\angle$ " represents 0 degrees from the real axis CCW, and any angle θ inside of it ($\angle \theta$) is CCW from the real axis.

So, A cosine wave reaches its maximum value at 0 degrees.

Similarly, a phasor at 0° degrees aligns ~~at~~ completely with the the real axis, maximizing its real world projection. Since both of them hit their maximum at zero, they form a perfect aligned baseline.

Sinusoid-Phasor Transformation

In a phasor diagram, frequency is not shown, as it is a static snapshot that shows only the starting angle and magnitude, but omits the continuous rotational speed. (frequency ω).

In frequency domain: By dropping time (t) from the equations, the math relies strictly on the system's frequency (ω) and the components' ~~at~~ fixed properties (like impedance). You are analyzing the circuit's steady state behavior at a specific frequency rather than calculating its moment-by-moment changes.

Derivatives become algebra!

In time domain, if you derive a cosine or sine function, you phase shift it by 90° degrees and multiply its magnitude by ω .

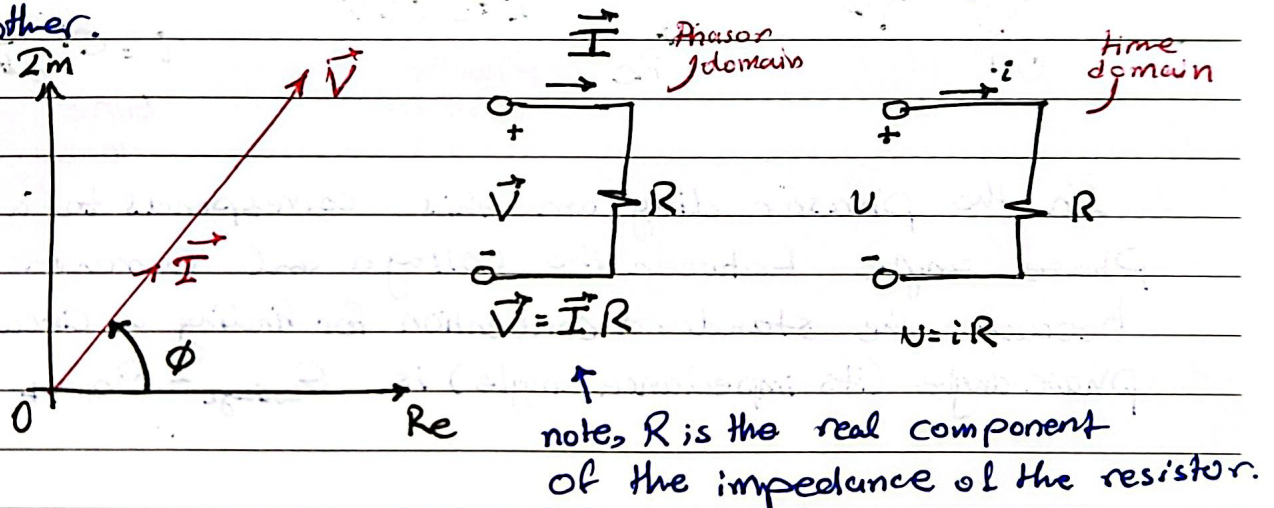
In frequency domain, you can just multiply by (j) to rotate a vector 90° counter clock-wise.

So, to achieve the effect of $\frac{dV}{dt}$, we multiply V by ω and j

$$\begin{array}{ccc} \frac{dV}{dt} & \longleftrightarrow & j\omega V \\ \text{time domain} & & \text{Phasor domain} \end{array}$$

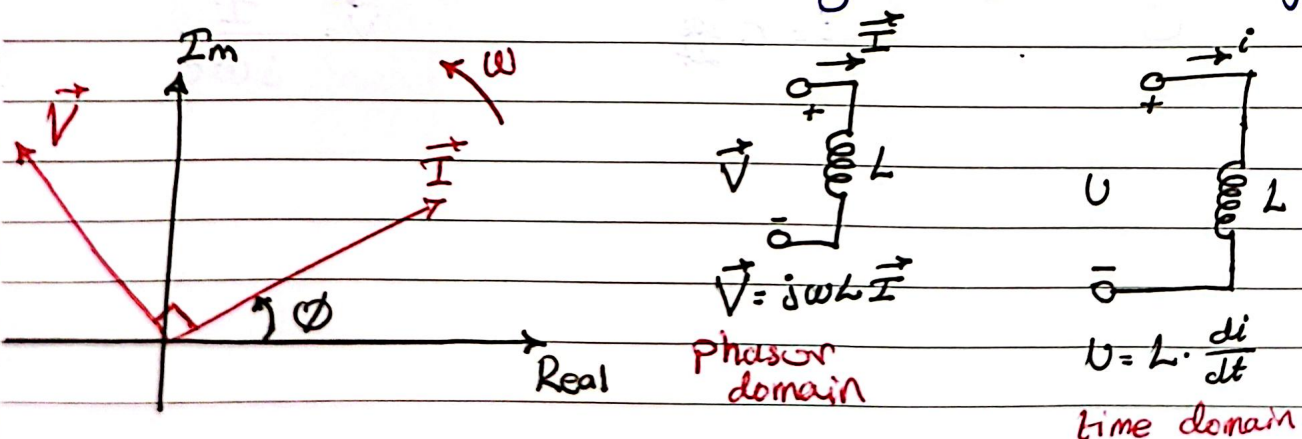
Phasor Relationships for Resistors

- Every circuit element has a relationship between its current and voltage.
- And these relationships can be mapped into phasor relationships, for resistors, capacitors and inductors.
- For the resistor, the voltage and current are related via Ohm's law
- For the resistor, both voltage and current are in-phase with each other.



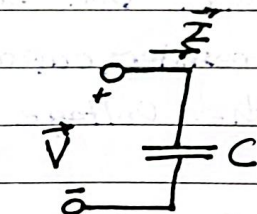
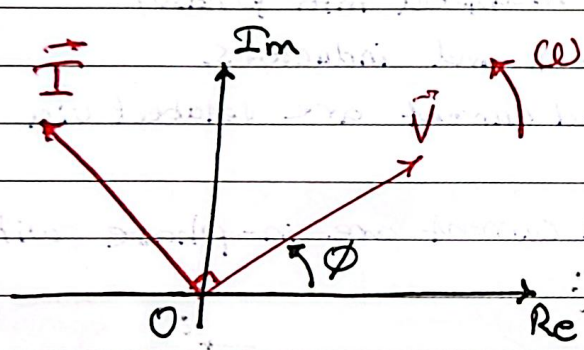
Phasor relationships for Inductors

- The ~~resistor~~ inductor has a phase shift between the voltage across it and the current through it
- The voltage leads the current by 90° (the current lags the voltage)



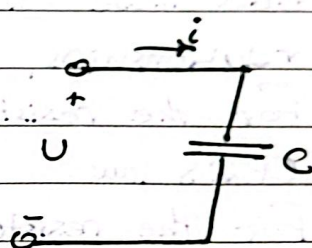
Phasor Relationships for Capacitors

- Capacitors have an opposite phase relationship to the inductors.
- The current leads the voltage (the voltage lags the current)



$$\vec{I} = j\omega C \vec{V}$$

Phasor domain



$$i = C \cdot \frac{du}{dt}$$

time domain

- In the phasor diagram this corresponds to a negative phase angle between the voltage and current because the standard convention for finding a circuit's phase angle (its impedance angle) is $\theta_{\text{voltage}} - \theta_{\text{current}}$.

Voltage current Relationships

Element	Time Domain	Frequency domain
R	$U = R i$	$\vec{V} = R \vec{I}$
L	$U = L \frac{di}{dt}$	$\vec{V} = j\omega L \vec{I}$
C	$i = C \frac{du}{dt}$	$\vec{V} = \frac{\vec{I}}{j\omega C}$