

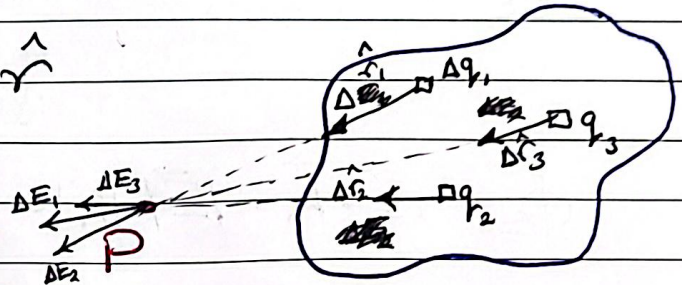
Chapter 23

Continuous Charge Distributions and Gauss's Law

electric field of a continuous charge distribution

$$\vec{E} = k_e \int \frac{dq}{r^2} \hat{r}$$

if we took Δq
very small then
we integrate
and put dq



Charge Density

$$\rho = \frac{Q}{V}$$

Volume
charge
density

$$dq = dV \cdot \rho$$

$$\sigma = \frac{Q}{A}$$

Surface
charge
density

$$dq = \sigma dA$$

$$\lambda = \frac{Q}{l}$$

Linear charge
density

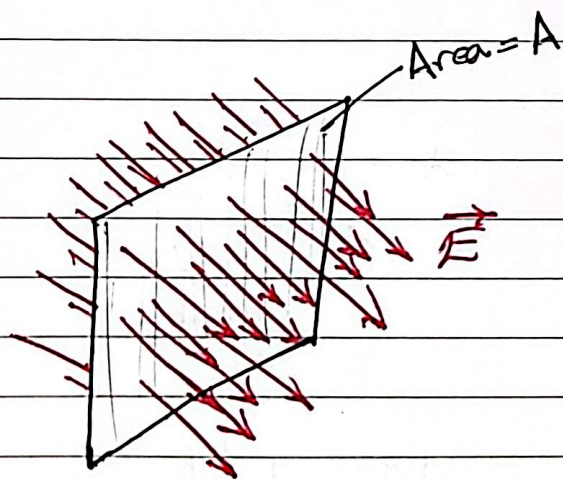
$$dq = \lambda dl$$

3-2-2020

Electric Flux

$\Phi_E = EA \cos \theta$ ← flux is just a measure of how much of the electric field is going through a specific area.

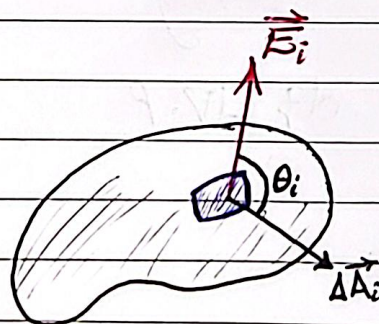
$$\Phi_E = \vec{E} \cdot \vec{A} = |\vec{E}| \cdot |\vec{A}| \cos \theta$$



$$\Phi_{E,i} = E_i \Delta A \cos \theta_i = \vec{E}_i \cdot \Delta \vec{A}_i$$

$$\Phi_E \approx \vec{E}_i \cdot \Delta \vec{A}_i$$

$$\Phi_E = \int_{\text{surface}} \vec{E} \cdot d\vec{A}$$



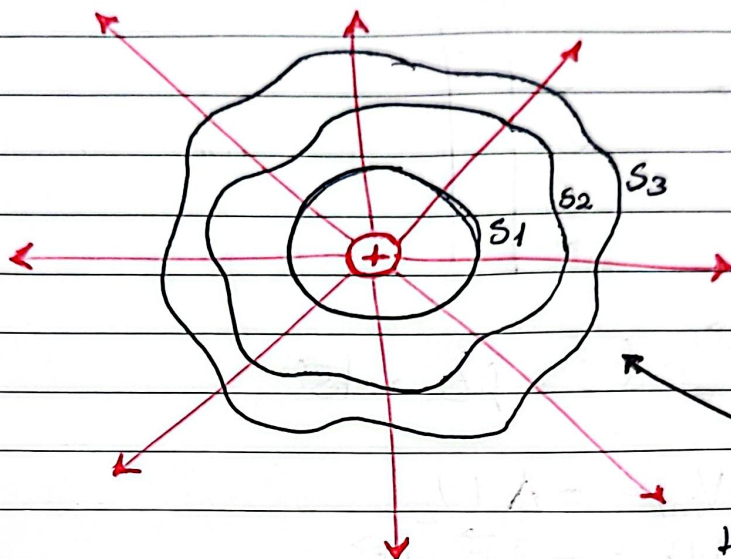
↑
the Electric field $\vec{E}_i$ makes an angle θ_i with the vector $\Delta \vec{A}_i$, which is defined as being normal to the surface element.

The Electric Flux over a closed ~~surface~~ surface (e.g. sphere)

$$\Phi_E = \oint \vec{E} \cdot d\vec{A}$$

Gaussian Surface and Flux

Net flux through any closed surface surrounding a point charge q is given by $\frac{q}{\epsilon_0}$ and is independent of the shape of the surface.

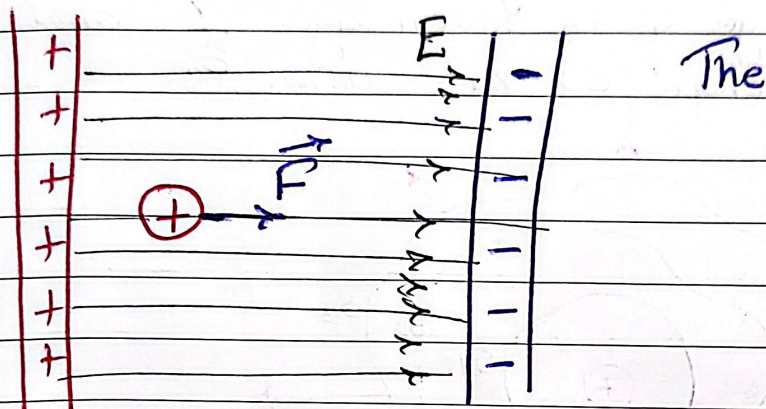


the net electric flux is the same through all surfaces

$$\Phi_{S_1} = \Phi_{S_2} = \Phi_{S_3}$$

A logical explanation to this is that when you change the bending of the surface at a ~~certain~~ certain point, ~~if~~ if you bend it inward towards the charge the overall area gets smaller and the electric field gets bigger so the ~~whole~~ whole effect is cancelled out and the net flux doesn't change.

Electric potential and potential Difference



$$W_{int} = \vec{F}_e \cdot d\vec{s} = q\vec{E} \cdot d\vec{s}$$

the work
the electric field does

$$W_{int} = -\Delta U_E$$

$$dU_E = -W_{int} = -q\vec{E} \cdot d\vec{s}$$

$$U_E = -q \int_A^B \vec{E} \cdot d\vec{s}$$

$$V = \frac{U_E}{q}$$

$$W = q\Delta V$$

difference
in electric
potential

$$\Delta V = \frac{\Delta U_E}{q} = - \int_A^B \vec{E} \cdot d\vec{s}$$

Electric potential

energy } $(1\text{eV}) = 1.6 \times 10^{-19} \text{ J}$

$$1 \text{ e.v.} = 1.6 \times 10^{-19} \text{ C} \cdot \frac{\text{J}}{\text{C}} = 1 \cdot \text{J} = [\text{J}]$$

$$\Delta V = - \int_A^B \vec{E} \cdot d\vec{s}$$

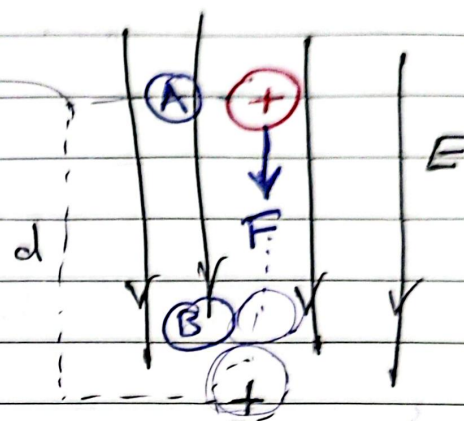
$$= - \int_A^B E ds \cos(0)$$

$$= - \int_A^B E ds$$

$$= - E(B-A)$$

$$\Delta V = - E(d_f - d_A)$$

$$\boxed{\Delta V = -Ed}$$



$$\Delta U_E = q\Delta V = -qEd$$

$$\boxed{\Delta U_E = -qEd}$$

~~$\Delta U_E = -qEd$~~

Electric Potential and Potential Energy due to point charges

$$\boxed{V = k_e \cdot \frac{q}{r}}$$

the sum of the voltage by charge q_i from a point P by distance r_i ,

$$\boxed{V = k_e \sum_i \frac{q_i}{r_i}}$$

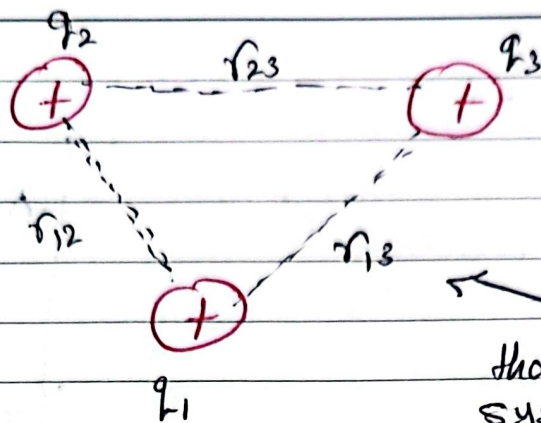
so this is the total voltage at a point P

$$W_{ext} = q_2 \Delta V$$

$$\boxed{U_E = k_e \cdot \frac{q_1 q_2}{r_{12}}}$$

This potential Energy is stored in the electric field between the two charges. It doesn't belong to one of them.

It is the energy needed to make that assembly of charges.



the potential energy that is stored in the system is given by:

$$U_E = k_e \left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right)$$

note:

The charges establish an electric potential as well as an electric field

$$\Delta U_E = W = q_2 \Delta V \rightarrow U_E - 0 = q_2 \left(k_e \cdot \frac{q_1}{r_{12}} - 0 \right)$$

$$U_E = k_e \cdot \frac{q_1 q_2}{r_{12}}$$

this is the Potential Energy difference between both charges from distance $r = \infty$ to r_{12}

So Voltage is how much potential energy ~~is~~ per coulomb of charge a charge is going to gain in a certain Electric field.

Potential Difference

~~Voltage~~ only appears in the presence of Electric fields.

Note: Best way to understand physics is to look at it from two points of view. From a common sense and physical part, and second from a mathematical point of view.

Obtaining the Value of the Electric field from the Electric potential.

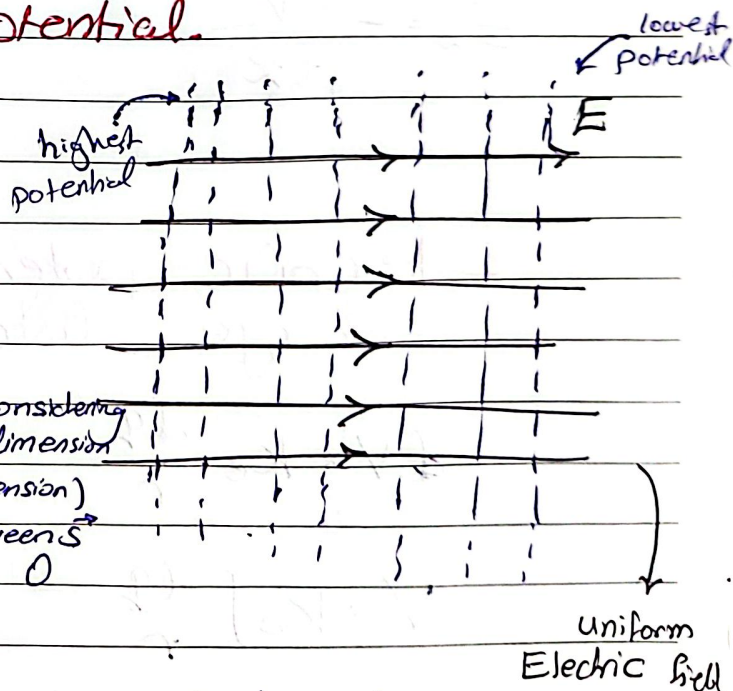
$$\Delta V = \int_A^B \vec{E} \cdot d\vec{s}$$

$$dV = - \vec{E} \cdot d\vec{s}$$

$E_x \cdot dx \rightarrow$ if we are considering only one dimension (the x dimension) and θ between $\vec{s}$ and $\vec{E}$ is 0

$$dV = - E_x dx$$

$$E_x = - \frac{dV}{dx}$$



So, the electric field in dimension x can be found by measuring how much the voltage changed over a distance in x dimension.

We can perform the same obtained rule (or relation) on other types of sources, like point charges and Electric dipole.

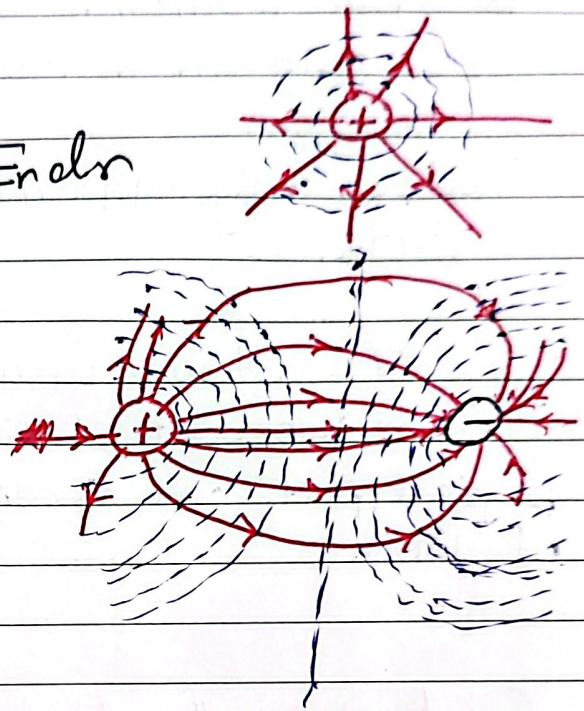
$$\vec{E} \cdot d\vec{S} = E_r \cdot dr \quad dV = -E_r dr$$

$$E_r = -\frac{dV}{dr}$$

$$E_x = -\frac{dV}{dx}$$

$$E_y = -\frac{dV}{dy}$$

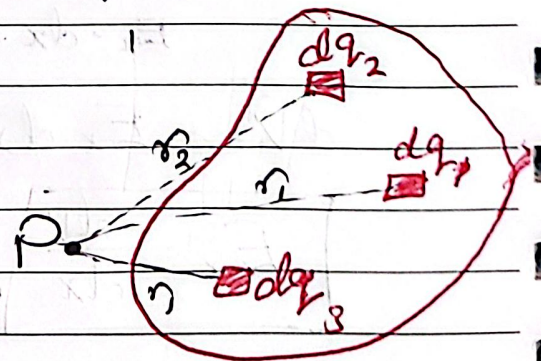
$$E_z = -\frac{dV}{dz}$$



Electric potential due to continuous charge distribution

$$dV = k_e \cdot \frac{dq}{r}$$

$$V = k_e \int \frac{dq}{r}$$

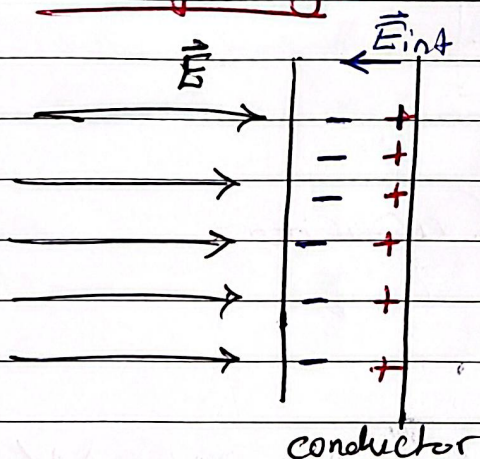


Electrostatic Equilibrium

Electrostatic Equilibrium is when charges have stopped moving and are stationary, and if any net charge is placed on the conductor, the charges will reside on the surface. Why? Because ~~Electrons~~ ^{charges} are ~~are~~ free to move, so the surface is the best place for least repulsion.

In a charged conductor, the Voltage inside the conductor is constant, and the Electric field = 0

Faraday's cage

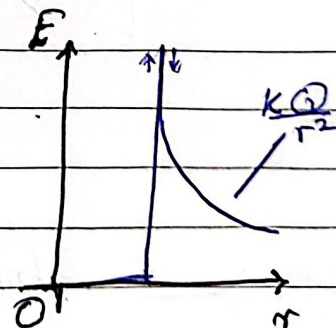
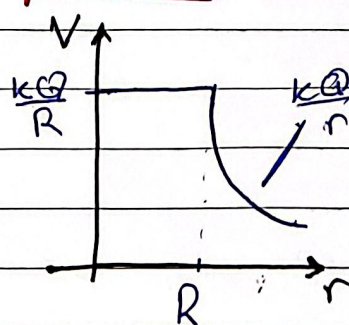
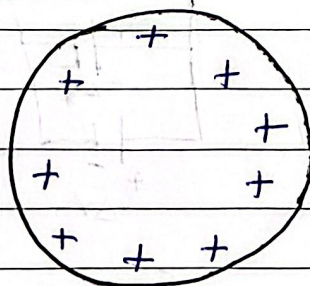


$$\vec{E}_{int} = -\vec{E}$$

So E_{net} on the conductor will equal zero

Note: A charged conductor would still work as a Faraday cage

Electrostatic Equilibrium - 2



Chapter 25

Capacitance and Dielectrics

17/Feb/2026

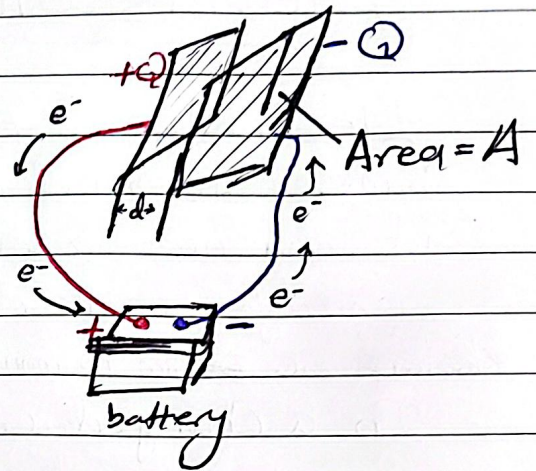
$$Q = C \Delta V$$

$$C = \frac{\epsilon_0 A}{d}$$

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$

$$\Delta V = Ed = \frac{Qd}{\epsilon_0 A}$$

$$C = \frac{Q}{\Delta V} = \frac{Q}{\frac{Qd}{\epsilon_0 A}} = \frac{\epsilon_0 A}{d} \quad \boxed{C = \frac{\epsilon_0 A}{d}}$$

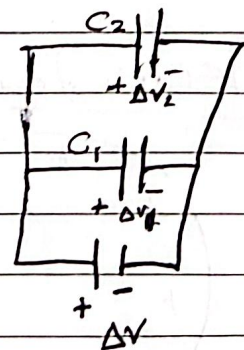


parallel combination of Capacitors

$$Q_{tot} = C_{eq} \Delta V$$

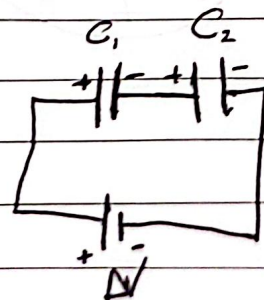
$$C_{eq} \Delta V = C_1 \Delta V_1 + C_2 \Delta V_2 \quad \Delta V = \Delta V_1 = \Delta V_2$$

$$\boxed{C_{eq} = C_1 + C_2}$$



Series Combination

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$



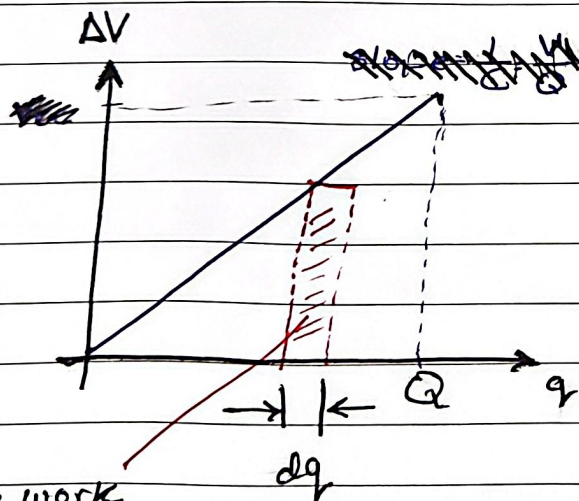
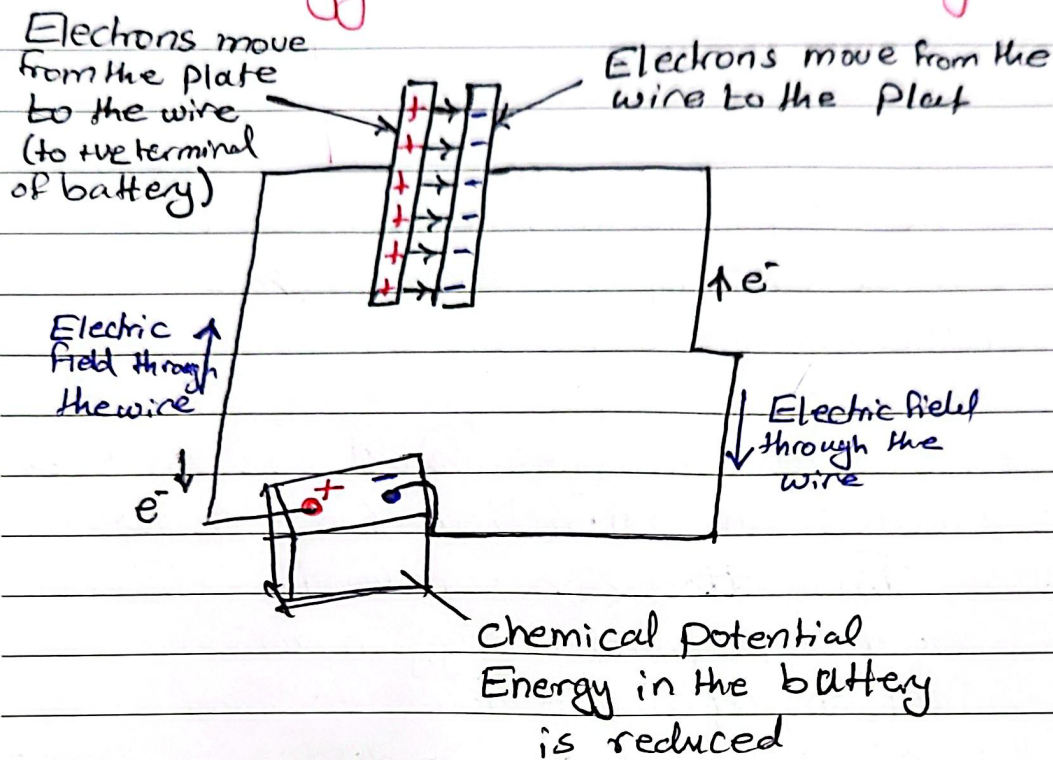
what is a capacitor?

A system that includes 2 conductors separated by a dielectric (e.g., vacuum). It serves storing electric potential energy in the electric field created between these conductors.

Because opposite charges build up on both plates, this creates an attraction force between both plates. This ~~attraction~~ constant attraction force is the electric potential energy in action. So both plates feel an attractive force toward each other, but the dielectric in the middle is ~~what~~ what is preventing both from touching.

Energy stored in a charged capacitor

20/2/2026



$$W = -\int \vec{r} \cdot \vec{E} d\vec{x} = -\int \vec{r} \cdot \frac{d\vec{V}}{d\vec{x}} d\vec{x}$$

↑ come from this

$$W = Q\Delta V$$

$$dW = \Delta V dq$$

$$W = \int_0^Q \frac{q}{C} dq = \frac{1}{C} \int_0^Q q dq = \frac{Q^2}{2C}$$

$$U_E = \frac{Q^2}{2C} = \frac{1}{2} Q\Delta V = \frac{1}{2} C(\Delta V)^2$$

the work required to move a charge dq through the potential difference ΔV across the capacitor plates is given approximately by the area of the shaded rectangle.

$$U_E = \frac{1}{2} \left(\frac{\epsilon_0 A}{d} \right) (Ed)^2 = \frac{1}{2} (\epsilon_0 A d) E^2$$

$$U_E = \frac{1}{2} \epsilon_0 E^2$$

A new concept arises here:

The conservation of charge, $Q_i = Q_f$, across the entire circuit.

Electric Dipole in an Electric Field

22/Feb/2026

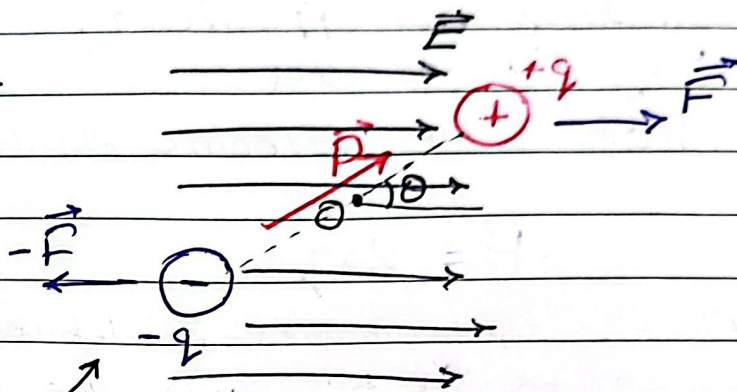
The direction of the electric field is opposite to the Electric Dipole

" $\vec{p}$ " is the dipole moment

$$p = 2aq$$

$$F \sin \theta$$

$$\tau = 2Fa \sin \theta$$



here we see that a torque exists around the origin between both charges

$$dW = \tau d\theta$$

$$\tau = pE \sin \theta$$

$$\begin{aligned} U_f - U_i &= \int_{\theta_i}^{\theta_f} \tau d\theta = \int_{\theta_i}^{\theta_f} pE \sin \theta d\theta = pE \int_{\theta_i}^{\theta_f} \sin \theta d\theta \\ &= pE [-\cos \theta]_{\theta_i}^{\theta_f} = pE (\cos \theta_i - \cos \theta_f) \end{aligned}$$

$$U_f = -pE \cos \theta$$

$$U_f = -\vec{p} \cdot \vec{E}$$

What is the Electric Dipole?

A configuration of two equal charges with equal magnitude and opposite sign, and are apart from each other by distance $2a$

What is the Electric dipole moment?

$$P = 2aq$$

Electric dipole moment is defined as the Vector directed from $-q$ toward $+q$ along the line between them and having magnitude of

$$P = 2aq$$

Dipole moment is a measure of how polarize a molecule take into consideration a point between the two charges

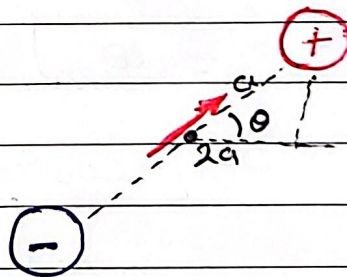
$$\tau = 2Fa \sin \theta$$

$$F = qE, P = 2aq$$

$$\tau = 2aqE \sin \theta = PE \sin \theta$$

$$\tau = PE \sin \theta$$

$$\vec{\tau} = \vec{P} \times \vec{E}$$



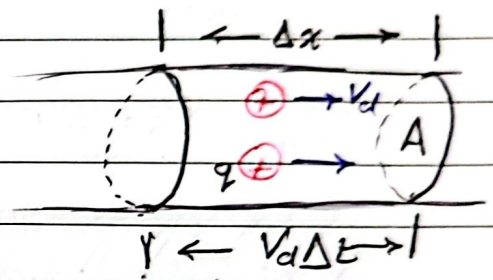
Current

$$I = \frac{dQ}{dt}$$

Drift Velocity

$$V_d = \frac{I}{nqA}$$

n : density of electrons
 q : amount of charge
 A : cross section area of wire



$$\Delta Q = (nA\Delta x)q$$

$$= (nAV_d\Delta t)q$$

ΔQ is the amount of charge that is passing through the cross-sectional area.

$$I_{avg} = \frac{\Delta Q}{\Delta t} = nqV_dA$$

the last electrons that reach finish line at time t

leader electron that cross the finish line at $t=0$

considering only one electron

$$d = Ut \rightarrow \Delta x = V_d t$$

in a time Δt , all electrons will move displacement Δx .

In that cylinder, what is the number of electrons?

$$N = A\Delta x \times \text{Density}$$

Density: n electrons/ m^3

total charge of these electrons

$$Q = A\Delta x n \cdot q$$

q : one electron charge (e)

$$\Delta x = V_d \Delta t$$

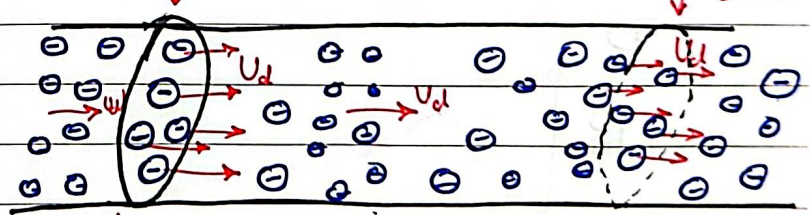
$$Q = AV_d \Delta t n \cdot q$$

$$I = \frac{Q}{t} = \frac{AV_d \Delta t n \cdot q}{t}$$

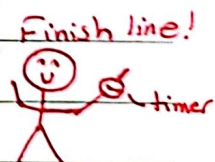
Note: $q = e^-$ to get drift velocity of electrons

$$I = AnqV_d$$

$$V_d = \frac{I}{Anq}$$



All electrons will move displacement Δx in time Δt



Chapter 27: Direct Current Circuits

Electromotive force

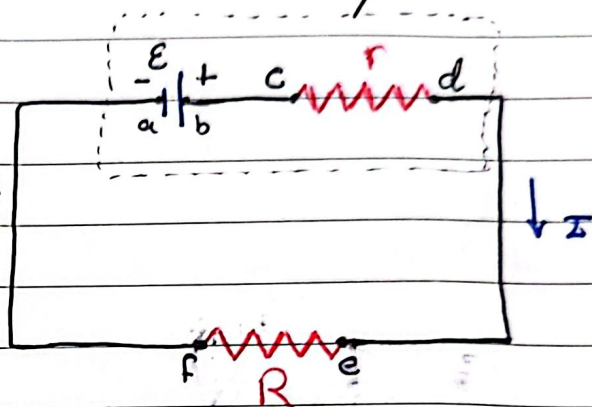
$$\Delta V = \mathcal{E} - Ir$$

Voltage
difference
across R
 $\Delta V = IR$

$$\mathcal{E} = IR + Ir$$

$$I = \frac{\mathcal{E}}{R + r}$$

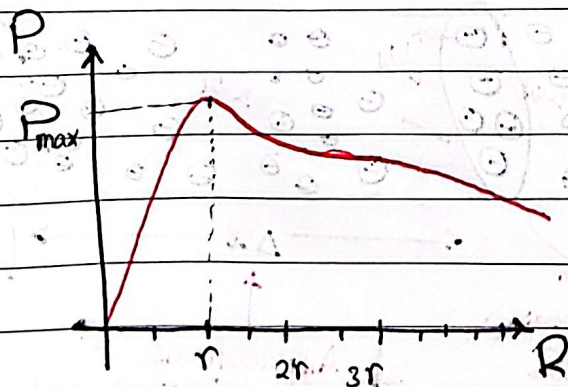
$I \uparrow$



$$I\mathcal{E} = I^2R + I^2r$$

~~if r is as small as possible, the maximum power is given to the load R .~~

- If you want the highest efficiency (percentage), so maximizing the percentage of power delivered to the external device, then you should make r as small as possible (minimize internal resistance).
- If you want the highest power (number) extracted from the battery, then you should make ~~sure~~ sure that load equals inner resistance ($R = r$).



Resistors in series and parallel

Series

$$R_{eq} = R_1 + R_2 + R_3 + \dots + R_n$$

Parallel

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_n}$$

Kirchhoff's Rules

Junction rule: At any junction, the sum of the currents must equal zero

$$\sum_{\text{junction}} I = 0$$

Loop rule: The sum of the potential differences across all elements around any closed circuit loop must be zero

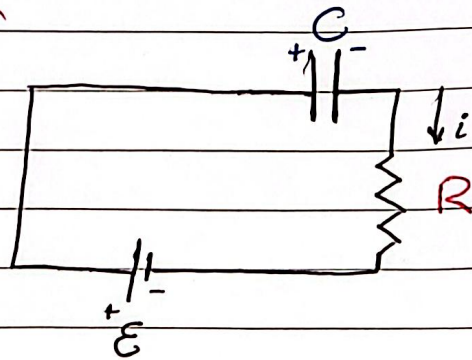
$$\sum_{\text{closed loop}} \Delta V = 0$$

Charging a capacitor

$$\mathcal{E} = V_{\text{capacitor}} + V_{\text{resistor}} = \frac{q}{C} + iR$$

KEC

$$Q_{\text{max}} = C\mathcal{E} \leftarrow \text{maximum charge}$$



$$i = \frac{dq}{dt} = \frac{\mathcal{E}}{R} - \frac{q}{RC}$$

$$\frac{dq}{dt} = \frac{\mathcal{E}}{R} - \frac{q}{RC} = \frac{\mathcal{E}C - q}{RC}$$

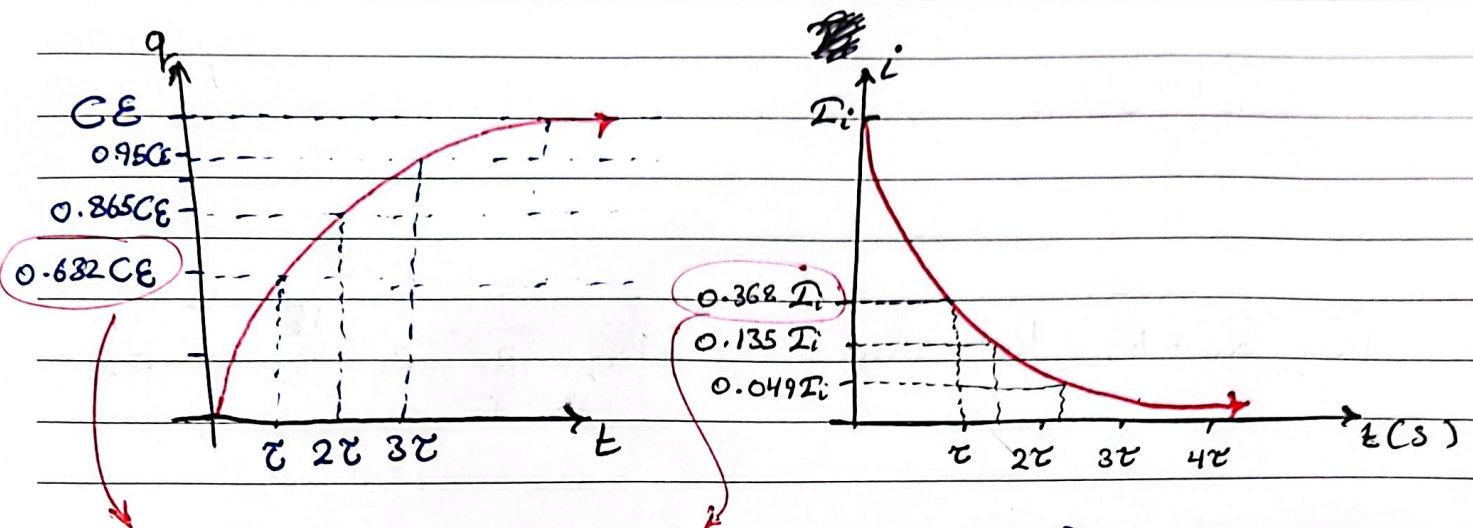
$$\tau = RC$$

$$\frac{dq}{q - C\mathcal{E}} = \frac{-1}{RC} dt \Rightarrow \int_0^q \frac{1}{q - C\mathcal{E}} dq = - \int_0^t \frac{1}{RC} dt$$

$$\ln \left(\frac{q - C\mathcal{E}}{-C\mathcal{E}} \right) = \frac{-t}{RC}$$

$$q(t) = C\mathcal{E} (1 - e^{-\frac{t}{RC}}) = Q_{\text{max}} (1 - e^{-\frac{t}{RC}})$$

$$i(t) = \frac{\mathcal{E}}{R} e^{-\frac{t}{RC}} \leftarrow i_{\text{max}}$$



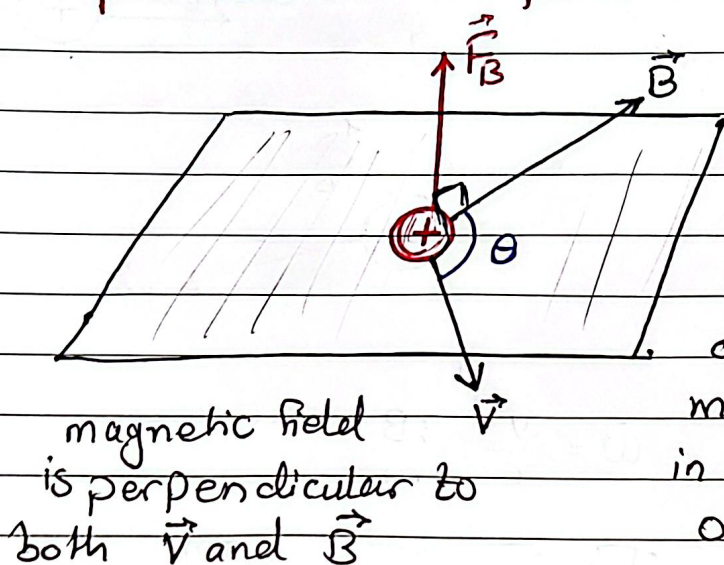
After a time interval of τ has passed, the charge is 68.2% of its maximum value CE

36.8% of I_i (max current) is the current after one time constant Period has passed

Chapter 28 - Magnetic Fields

14/3/2026

A particle in a field (Magnetic)



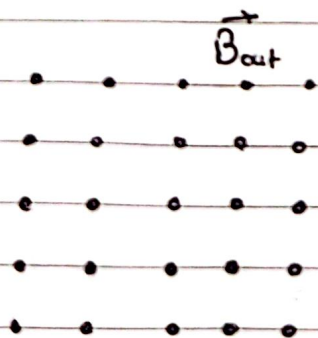
velocity of charge
external magnetic field magnitude and direction

$$\vec{F}_B = q(\vec{v} \times \vec{B})$$

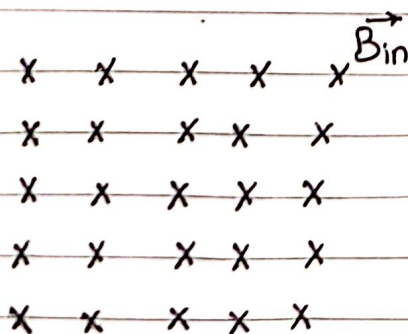
Note: The magnetic forces on oppositely charged particles moving ~~at the~~ the same velocity in a magnetic field are in opposite directions.

Note: use the right hand rule for the +ve charges, and use the left hand rule for the -ve charges. Because q for -ve charge will be negative, hence $\vec{F}_B$ will have an opposite direction than +ve charge.

The magnetic force only acts on a charged particle in a magnetic field when the charge is in motion ($\vec{v} \neq 0$). Why? I think because when a charge is moving, it has generated around it a magnetic field, and that magnetic field interacts with the external magnetic field ($\vec{B}$).

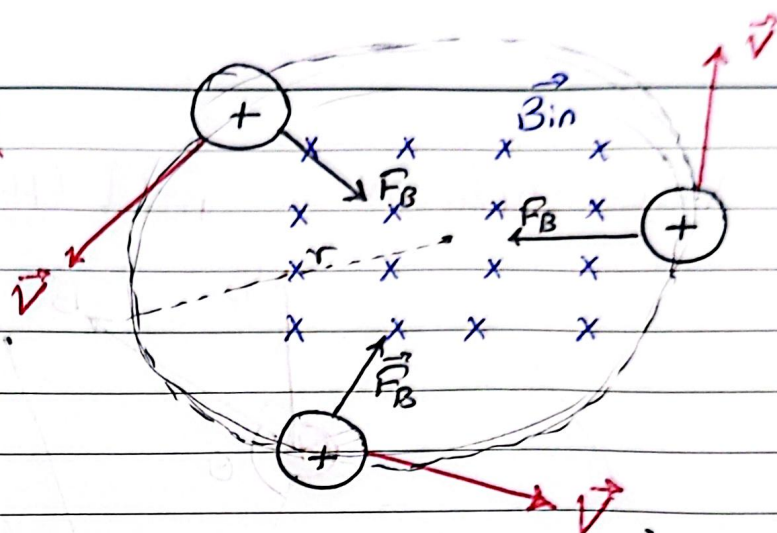


magnetic field lines going out the paper



magnetic field lines going in the paper

Motion of a charged particle in a uniform magnetic field



$$\Sigma F = F_B = ma$$

$$F_B = qvB = \frac{mv^2}{r}$$

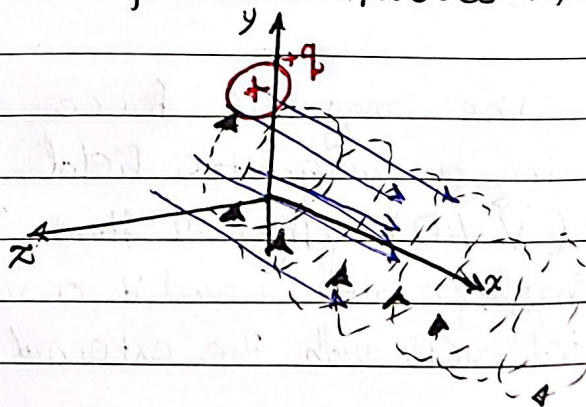
$$r = \frac{mv}{qB}$$

$$\omega = \frac{v}{r} = \frac{qB}{m}$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{qB}{m}} = \frac{2\pi m}{qB}$$

Note: if θ between $\vec{v}$ and $\vec{B}$ is exactly 90° , the charged particle moves in a perfect circle. If $\theta \neq 90^\circ$, then the particle moves in a helical path.

$$v = \sqrt{v_y^2 + v_z^2}$$



Applications involving charged particles moving in a magnetic field

- Velocity selector
- Mass spectrometer
- The Cyclotron

Velocity selector

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

$$q\vec{E} = qv\vec{B}$$

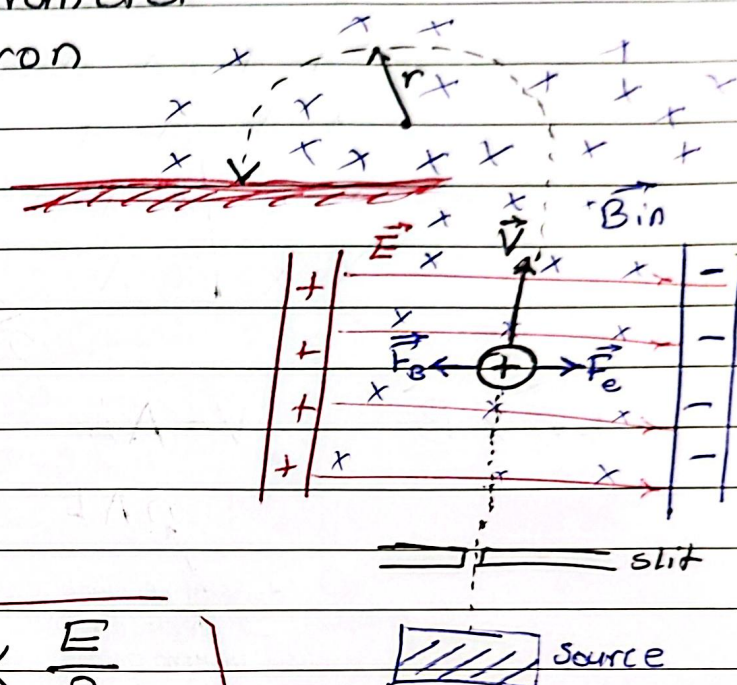
$$v = \frac{E}{B}$$

$$v > \frac{E}{B}$$

deflect to the left

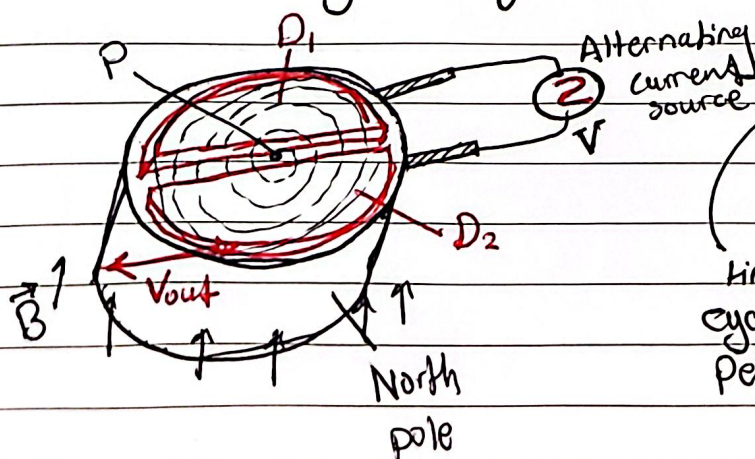
$$v < \frac{E}{B}$$

deflect to the right



you can use it to separate ions according to mass-to-charge ratio (usually to separate isotopes).

The Cyclotron: This speeds up a charge particle without needing a long tube.

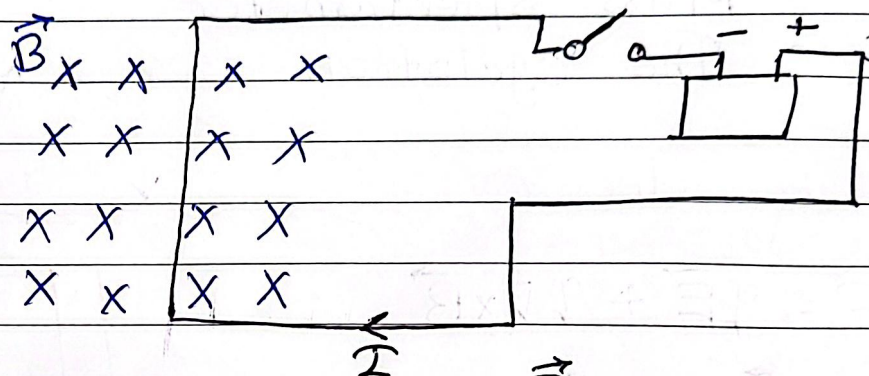


$$T = \frac{2\pi m}{qB} \quad v = \frac{qBR}{m}$$

$$K = \frac{1}{2}mv^2 = \frac{q^2 B^2 R^2}{2m}$$

time period of particle completing a full cycle (circle) is in sync with the time period of the AC source time period

Magnetic Force acting on a current carrying conductor



$$q \vec{V}_d \times \vec{B}$$

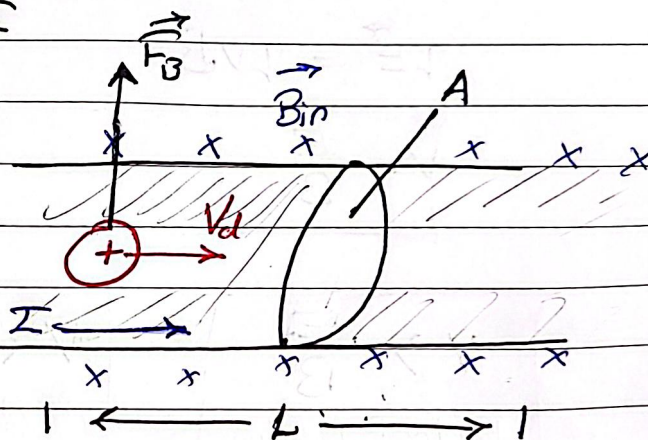
$$V = AL$$

charges in segment: nAL

density of number of electrons in volume

$$I = nqV_dA$$

$$\vec{F}_B = (q\vec{V}_d \times \vec{B})nAL$$



a scalar to include all electrons in the wire

$$\vec{F}_B = I \vec{L} \times \vec{B}$$

Current magnitude

wire length and in direction of current

magnetic field strength

$$d\vec{F}_B = I d\vec{s} \times \vec{B}$$

$$\vec{F}_B = I \int_a^b d\vec{s} \times \vec{B}$$

Force on a loop of conductor

force on semi-circle:

$$\vec{F}_1 = I \int_a^b d\vec{s} \times \vec{B} = I \int_{-R}^R B dx \hat{k}$$

$$= \boxed{2IRB\hat{k}}$$

force on straight wire:

$$d\vec{F}_2 = I d\vec{s} \times \vec{B} = -IB \sin\theta ds \hat{k}$$

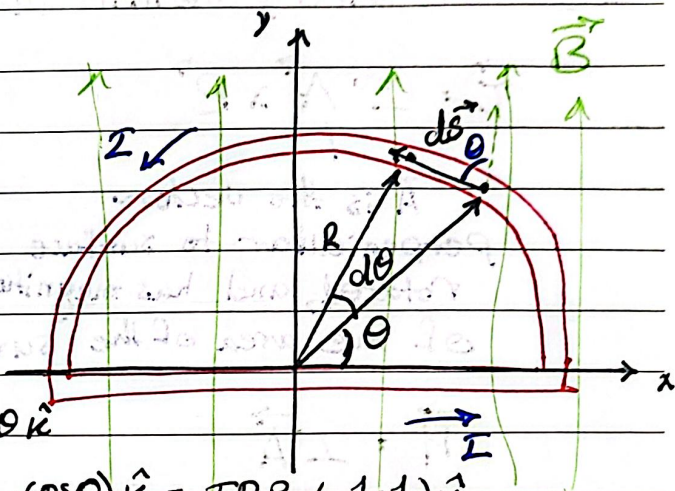
$$ds = R d\theta$$

$$\vec{F}_2 = \int_0^\pi -IRB \sin\theta d\theta \hat{k} = -IRB \int_0^\pi \sin\theta d\theta \hat{k}$$

$$= -IRB [-\cos\theta]_0^\pi \hat{k} = IRB (\cos\pi - \cos 0) \hat{k} = IRB (-1 - 1) \hat{k}$$

$$= \boxed{-2IRB\hat{k}}$$

$$F_{net} = 2IRB\hat{k} - 2IRB\hat{k} = 0 \quad \boxed{F_{net} = 0 \text{ N}}$$

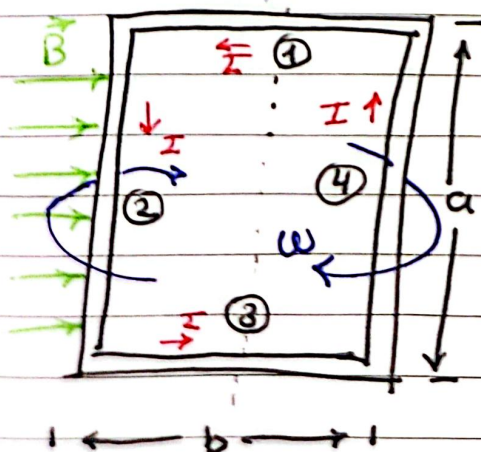


Note: Net force of magnetic field on a ~~wire~~ closed loop of conductor is always zero

Note: Though there is no net Force, there exists a torque that acts on the wire. net force causes translational motion, while torque causes rotational motion.

Torque on a current loop in a Uniform magnetic field

no magnetic forces on sides 1 and 3 due to being parallel with magnetic field

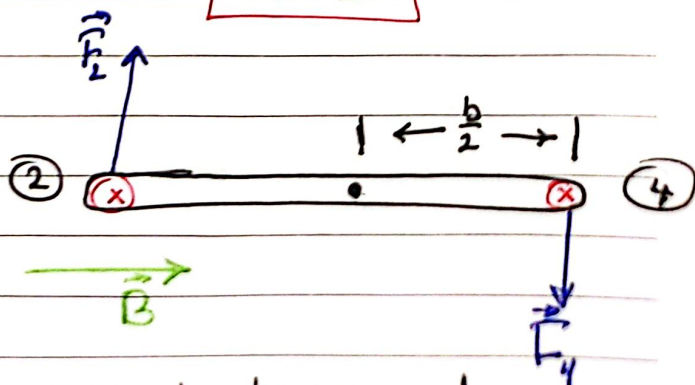


$$F_2 = F_4 = I a B$$

$$\tau_{max} = F_2 \cdot \frac{b}{2} + F_4 \cdot \frac{b}{2}$$

$$= (IaB) \cdot \frac{b}{2} + (IaB) \cdot \frac{b}{2}$$

$$= \boxed{IabB}$$

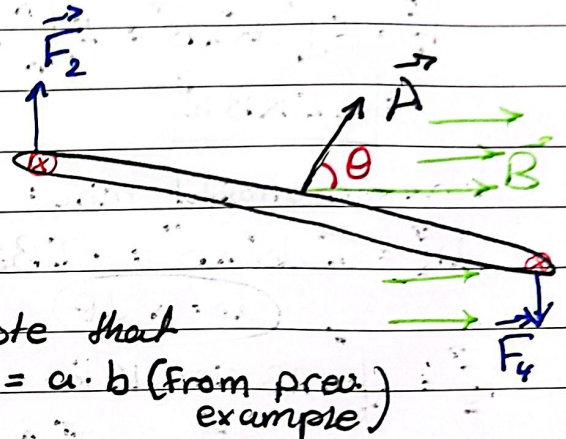


Note: The loop will oscillate back and forth. It won't rotate in a continuous loop

Instead of calculating the force on every segment of the wire, we introduce magnetic dipole moment.

$$\vec{\tau} = I \vec{A} \times \vec{B}$$

$\vec{A}$ is the vector perpendicular to surface rotated, and has magnitude of the area of the surface



note that $A = a \cdot b$ (from prev. example)

$$\vec{M} = I \vec{A}$$

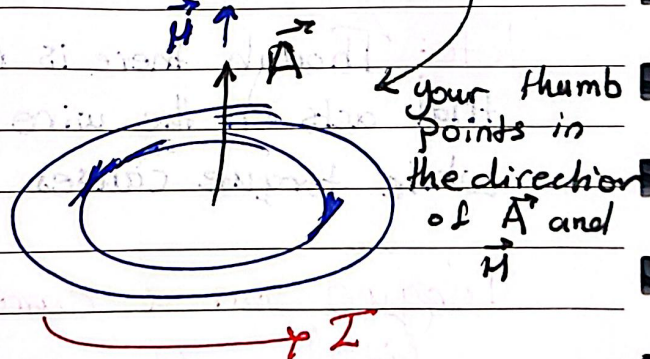
$$\vec{M}_{\text{coil}} = N I \vec{A}$$

number of turns

$$\vec{\tau} = \vec{M} \times \vec{B}$$

→ similar to electric dipole torque
 $\vec{\tau} = \vec{p} \times \vec{E}$

$$U_B = - \vec{M} \cdot \vec{B}$$



Chapter 29: Sources of Magnetic Field

Essentially the magnetic equivalent of Coulomb's law $E = k_e \frac{q}{r^2} \hat{r}$

$$d\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{I d\vec{s} \times \vec{r}}{r^2}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A} \quad \leftarrow \text{in a vacuum}$$

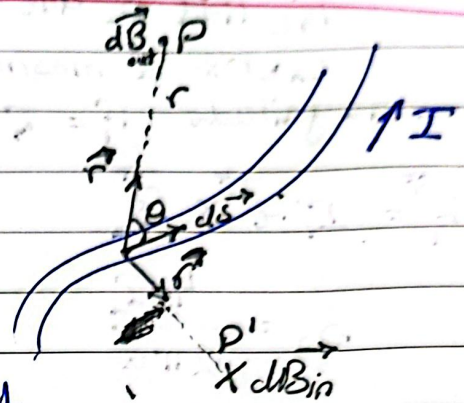
if we assume I is constant

magnetic field generated at a point from the wire due to current $\rightarrow \vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{s} \times \vec{r}}{r^2}$

All the magnetic field vectors tangent to the wire sum to make a total magnetic that is like a circle around the wire.

The magnetic field due to a thin straight wire:

$$B = \frac{\mu_0 I}{2\pi a}$$



The magnetic force between two parallel conductors

26-3-2026

The field $\vec{B}_2$ due to the current in wire 2 exerts a magnetic force of magnitude $F_1 = I_1 l B_2$ on wire 1

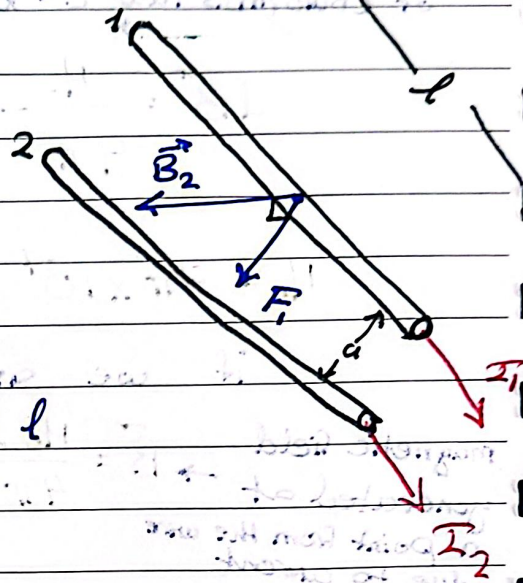
$$\vec{F}_1 = I_1 \vec{l} \times \vec{B}_2 \quad F_1 = I_1 l B_2$$

$$B_2 = \frac{\mu_0 I_2}{2\pi a}$$

$$F_1 = I_1 l \cdot \left(\frac{\mu_0 I_2}{2\pi a} \right) = \frac{\mu_0 I_1 I_2}{2\pi a} l$$

$$F_1 = \frac{\mu_0 I_1 I_2}{2\pi a} l$$

$$\frac{F_B}{l} = \frac{\mu_0 I_1 I_2}{2\pi a}$$

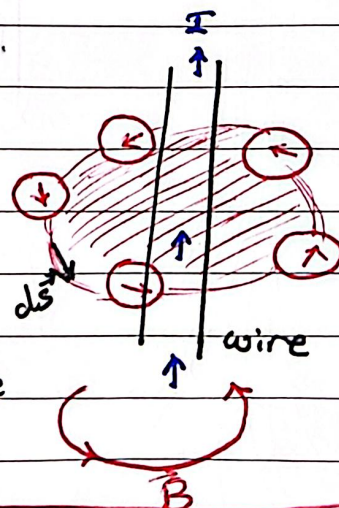


Ampere's Law

$$\oint \vec{B} \cdot d\vec{s} = B \oint ds = \frac{\mu_0 I}{2\pi r} \cdot (2\pi r) = \mu_0 I$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{enclosed}}$$

ds is a tiny step in the circular path of the magnetic field (or just a loop around the wire)



The line integral of $\vec{B} \cdot d\vec{s}$ of a wire carrying current I equals $\mu_0 I$.

Ampere's law states that the line integral of a magnetic field (B) around any closed path is directly proportional to the net electrical current enclosed by the path

Gauss's law is about total charge enclosed in a loop. Ampere's law is about total current enclosed.

The magnetic field created by a long current-carrying wire

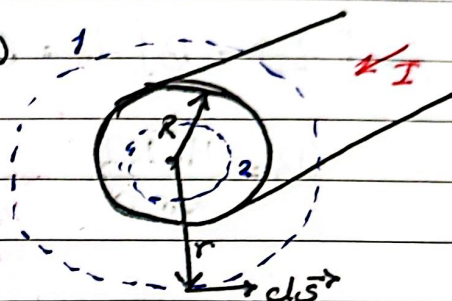
$$\int \vec{B} \cdot d\vec{s} = I \mu_0 \quad \vec{B} \cdot d\vec{s} = B \cdot ds \cdot \cos(0) = B \cdot ds$$

$$\int \vec{B} \cdot d\vec{s} = B \int ds = B \cdot s = B(2\pi r) = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

for $r \geq R$

for loop 2



current I in loop 2 is not the same as current I in loop 1

$$I_1 > I_2$$

as current scales with area as current is uniformly distributed.

$$\frac{I'}{I} = \frac{A'}{A} = \frac{\pi r^2}{\pi R^2}$$

$$I' = \frac{r^2}{R^2} I$$

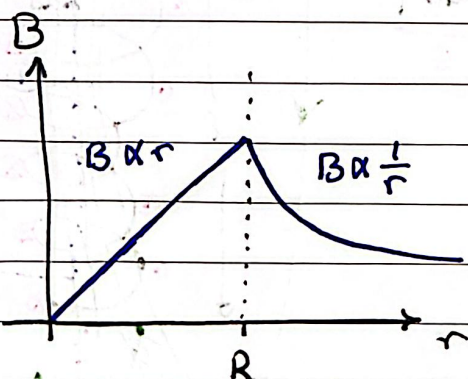
$$\int B \cdot ds = \mu_0 I'$$

$$B(2\pi r) = \mu_0 \cdot \frac{r^2}{R^2} I$$

$$B = \frac{\mu_0 r^2}{2\pi R^2} I = \frac{\mu_0 r}{2\pi R^2} I$$

$$B = \frac{\mu_0 r}{2\pi R^2} I$$

for loop 1 $r < R$



$$B = \begin{cases} \frac{\mu_0 I}{2\pi r}, & (\text{for } r \geq R) \\ \frac{\mu_0 I}{2\pi R^2} r, & (\text{for } r < R) \end{cases}$$

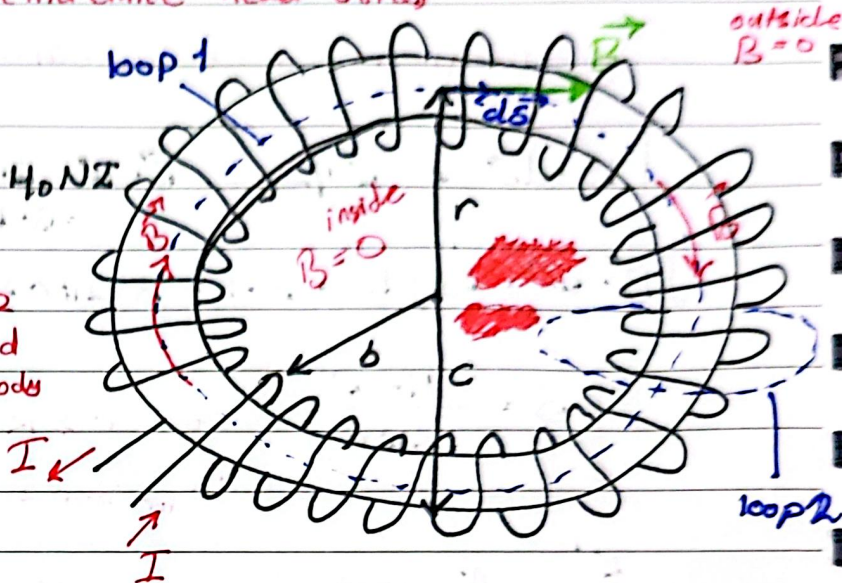
Toroid - the ambulance (machine head voice)

for loop 1

$$\int \vec{B} \cdot d\vec{s} = B \int ds = B(2\pi r) = \mu_0 N I$$

$$B = \frac{\mu_0 N I}{2\pi r}$$

← for loop 2
inside toroid
circular body



~~the magnetic field inside the toroid loop is the most focused~~

~~NO magnetic field inside the toroid loop is zero, by ampere's law. Also outside the toroid.~~

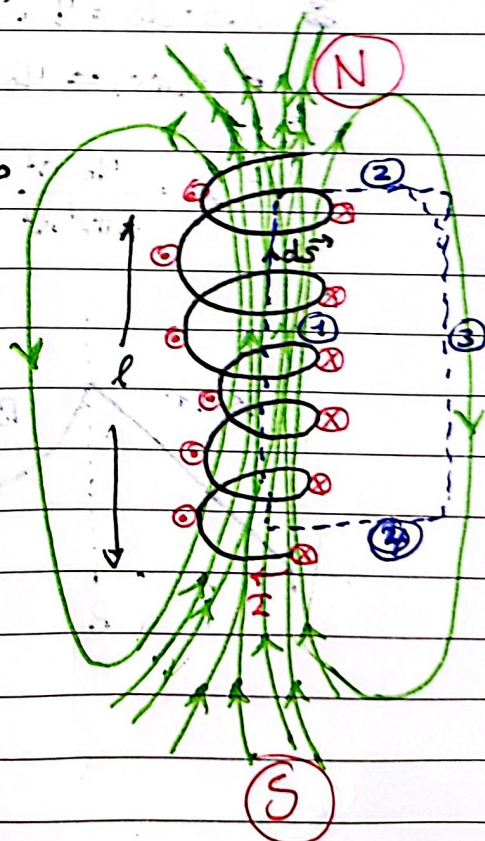
The magnetic field of a solenoid

$$\oint \vec{B} \cdot d\vec{s} = \underbrace{\oint_{\text{Path 1}} \vec{B} \cdot d\vec{s}}_0 + \underbrace{\oint_{\text{Path 2}} \vec{B} \cdot d\vec{s}}_0 + \underbrace{\oint_{\text{Path 3}} \vec{B} \cdot d\vec{s}}_{\approx 0} + \underbrace{\oint_{\text{Path 4}} \vec{B} \cdot d\vec{s}}_0$$

$$= \int_{\text{Path 1}} \vec{B} \cdot d\vec{s} = N I \mu_0$$

$$= B l = N I \mu_0$$

$$B = \frac{N I \mu_0}{l}$$

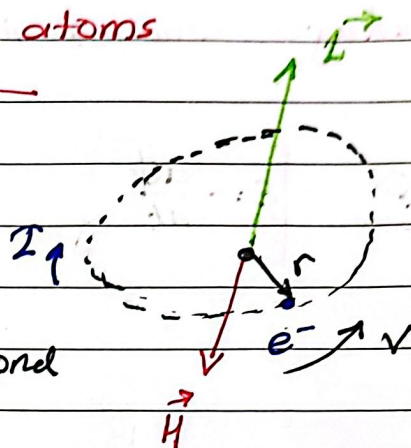


The magnetic moments of atoms

$$\vec{M} = I\vec{A} \quad \vec{L} = m_e \vec{r} \times \vec{v}$$

$$\vec{L} = m_e v r \leftarrow \text{angular momentum}$$

magnetic moment is directly proportional to angular momentum.



Note: $\vec{A}$ is in the same direction as the magnetic field inside the loop (using right hand rule) (you don't pick its direction!)

$$\vec{M} = -\frac{e}{2m_e} \vec{L}$$

$\frac{e}{2m_e}$ is called the gyromagnetic ratio

derivation: $I = \frac{Q}{t} = \frac{e}{T} \leftarrow \text{time period}$ $T = \frac{2\pi r}{v}$

$$M = IA = \frac{e}{T} \cdot \pi r^2 = \frac{e}{\frac{2\pi r}{v}} \cdot \pi r^2 = \frac{ve}{2\pi} \cdot \pi r^2$$

$$M = \frac{vre}{2} \quad \text{note: } h = mvr$$

$$M = \frac{vre}{2} \cdot \frac{m_e}{m_e} = \frac{e \cdot (vr m_e)}{2m_e} = \frac{e}{2m_e} L$$

$$M = \frac{e}{2m_e} L$$

very important equation that connects electromagnetism and quantum physics atomic

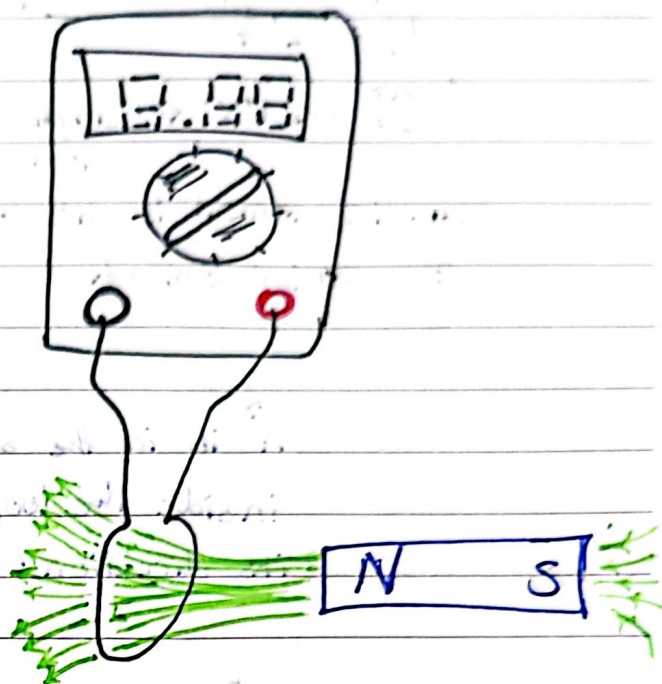
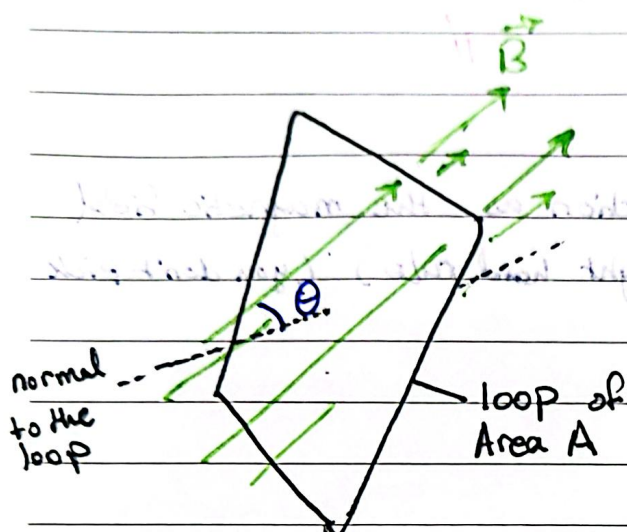
Chapter 30

Faraday's Law of induction

$$\mathcal{E} = -N \frac{d\phi}{dt}$$

number of turns

$$\mathcal{E} = - \frac{d\phi}{dt}$$



$$\mathcal{E} = - \frac{d}{dt} (BA \cos \theta)$$

Motional emf - What is the voltage difference across a wire moving in a magnetic field?

$$\Sigma \vec{F} = 0 \Rightarrow \vec{F}_E = \vec{F}_B$$

$$qE = qvB$$

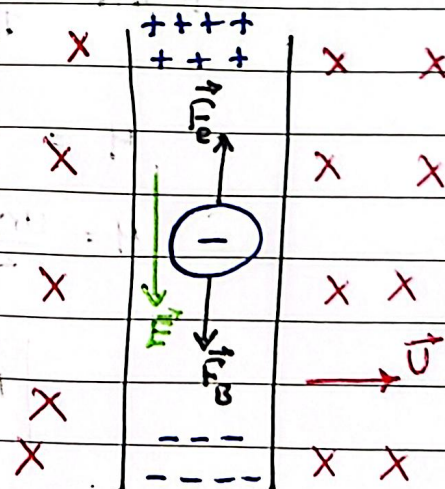
$$E = vB$$

$$\Delta V = Ed = vBd$$

$$\Delta V = B \ell v$$

$$\Delta V = B \ell v$$

they are equal once the charge resides in its charges region (F_B is first too strong compared to F_E , so when the charges reach their region $F_E = F_B$)
Velocity of the wire perpendicular to the magnetic fields



the voltage difference between the ends of the wire

$$\mathcal{E} = -B \ell v$$

$$I = \frac{B \ell v}{R}$$

Note: The magnetic force pushes the charges apart until the resulting electric force grows strong enough to perfectly balance it out, stopping the flow and creating a stable voltage

Motional emf induced in a rotating Bar

$$\mathcal{E} = B l v$$

$\mathcal{E}$ changes
as l changes

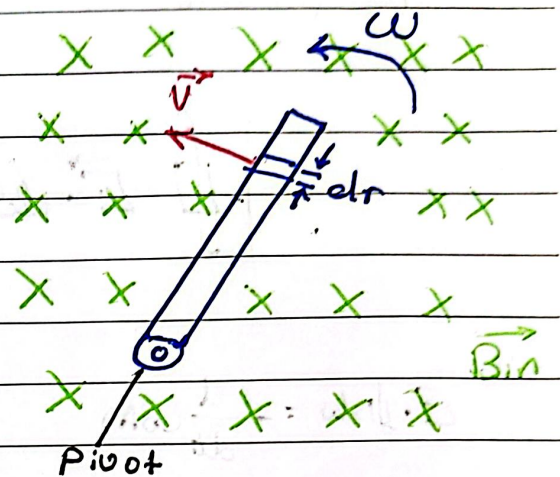
for each dr , because each dr
has a different speed

$$d\mathcal{E} = B v dr = B v(r) dr$$

$$v(r) = \omega r$$

$$\mathcal{E} = \int_0^l B v(r) dr = \int_0^l B \omega r dr$$

$$= B \int_0^l \omega r dr = \cancel{\frac{1}{2} B \omega l^2} \boxed{\frac{1}{2} B \omega l^2}$$



Lenz's law

The induced current in a loop is in the direction that creates a magnetic field that opposes the change in magnetic flux through the area enclosed by the loop

$$\boxed{\mathcal{E} = - \frac{d\Phi_B}{dt}}$$

Note: Faraday's law dictates the electrical voltage induced in any wire, while Lenz's law dictates the physical magnetic pushback that can only occur when a CLOSED loop allows an opposing current to flow.

The general form of Faraday's Law

$$\oint \vec{E} \cdot d\vec{s} = - \frac{d\Phi}{dt}$$

$$\vec{E} \cdot \oint d\vec{s} = - \frac{d}{dt}(BA)$$

$$E (2\pi r) = - \frac{dB}{dt} (\pi r^2)$$

$$E = - \frac{r}{2} \cdot \frac{dB}{dt}$$

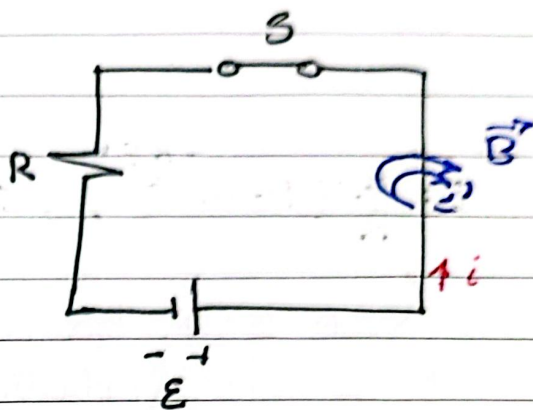
$$\Delta V = \int \vec{E} \cdot d\vec{s}$$

Chapter 31 - Inductance

14-04-2026

After the switch is closed, the current increases gradually (but fast) to its equilibrium

value. While the current is increasing, the magnetic field induced increases, and with it increases the magnetic flux through the loop, and hence an opposing emf is induced that opposes the original current!



Key point: while the current is increasing towards its equilibrium value, it faces something like a "resistance", which disappears once current gets to its equilibrium value. So, with that being said, we can say that ~~there~~ the induced magnetic field resists the change in current!

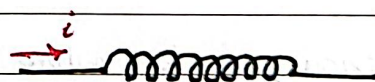
So, that resistance in the change in current is Inductance! (L)
what is the inductance for a solenoid?

$$\mathcal{E} = -N \cdot \frac{d\Phi_B}{dt} \quad B = \frac{\mu_0 N I}{l} \text{ (at the center of the solenoid)}$$

For a solenoid: $\Phi_B = BA = \mu_0 N i A / l$

$$\mathcal{E}_{\text{induced}} = -N \cdot \frac{d(\mu_0 N i A)}{dt} = -\frac{N^2 \mu_0 A}{l} \frac{di}{dt}$$

Inductance (L)



l comes from $B = \mu_0 n I = \frac{\mu_0 N I}{l}$

where n is the turn density, $n = \frac{N}{l}$ for the solenoid.

So, inductance L is just a constant

$$L = \frac{N^2 \mu_0 A}{l}$$

$$L = \frac{N \Phi_B}{i}$$

$$\mathcal{E}_L = -L \cdot \frac{di}{dt}$$

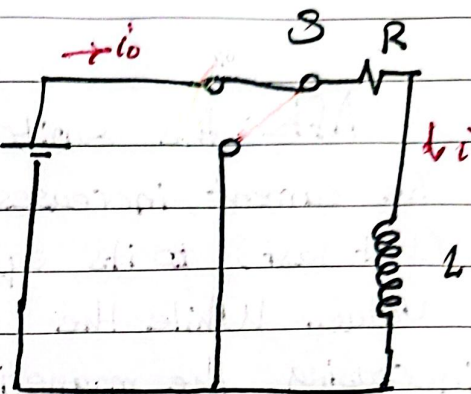
$$L = - \frac{\mathcal{E}_L}{\frac{di}{dt}}$$

← similar to $R = \frac{V}{I}$

Inductance is a measure of the opposition to a change in current.

RL Circuits

$$i = \frac{\mathcal{E}}{R} (1 - e^{-\frac{Rt}{L}}), \tau = \frac{L}{R}$$



at $t=0$, the current $i=0$, but by time gradually, the current increases to reach its equilibrium value.

So this equation shows us how current gradually builds up in the circuit.

how much is the current when $t = \tau$?

$$i = \frac{\mathcal{E}}{R} (1 - e^{-\frac{t}{\tau}}) = \frac{\mathcal{E}}{R} (1 - e^{-\frac{\tau}{\tau}}) = \frac{\mathcal{E}}{R} (1 - \frac{1}{e})$$

$$= \frac{\mathcal{E}}{R} (0.632) = 0.632 I_{\max}$$

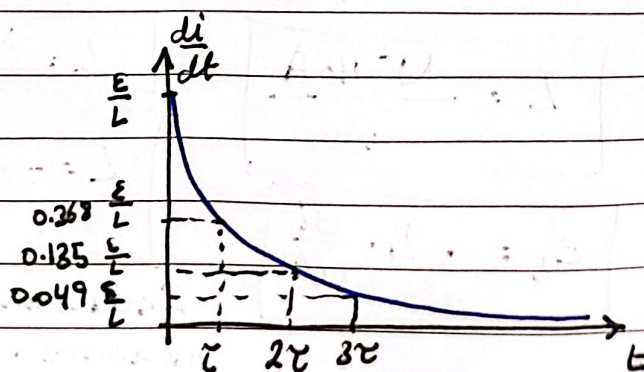
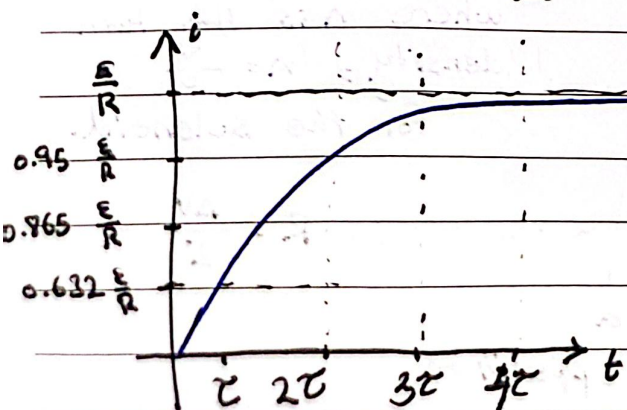
$$i = \frac{\mathcal{E}}{R} e^{-\frac{t}{\tau}} = I_i e^{-\frac{t}{\tau}}$$

this is when the inductor is in parallel with resistor without power source. the current decreases gradually in the circuit.

$$i(\tau) = 63.2\% I_{\max}$$

you know, L 's value is mostly small (in mH), so τ will be a very small number, meaning we won't really notice that build up in current.

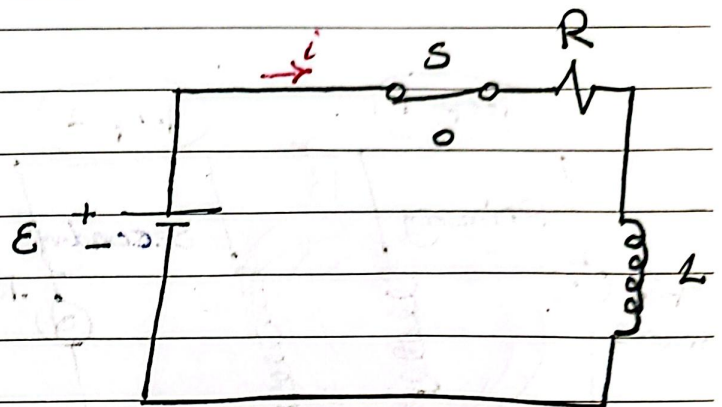
$$\text{as } t \rightarrow \infty, \frac{di}{dt} \rightarrow 0, \Rightarrow i = \frac{\mathcal{E}}{R} \quad \frac{di}{dt} = \frac{\mathcal{E}}{L} e^{-\frac{t}{\tau}}$$



Energy in a magnetic Field

$$P_{\text{battery}} = P_R + P_L$$

$$i\mathcal{E} = i^2 R + L i \frac{di}{dt}$$



$$P_L = \frac{dU_B}{dt} = \underbrace{L \cdot i}_{\mathcal{E}_L \cdot i} \frac{di}{dt}$$

~~Energy stored in an inductor~~

$$dU_B = L i di$$

$$\int_0^{i(i)} dU_B = \int_0^i L i di$$

$$U_B(i) = \frac{1}{2} L i^2$$

← energy stored in an inductor (as a magnetic field)

Magnetic energy Density in a solenoid

As we said energy in an inductor is stored in the magnetic field. But how dense is that energy? how much energy is distributed in unit volume?

$$L = \mu_0 n^2 V$$

turn density
interior of the solenoid
interior

it came from $L = \frac{N^2 \mu_0 A}{l} \cdot \frac{l}{l} V$

$$= \left(\frac{N^2}{l^2}\right) \cdot \mu_0 V = n^2 \mu_0 V$$

turn density squared

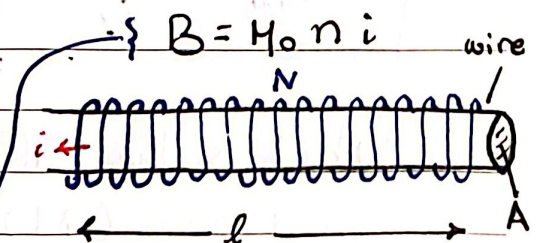
$$U_B = \frac{1}{2} L i^2$$

represent the energy in terms of magnetic field

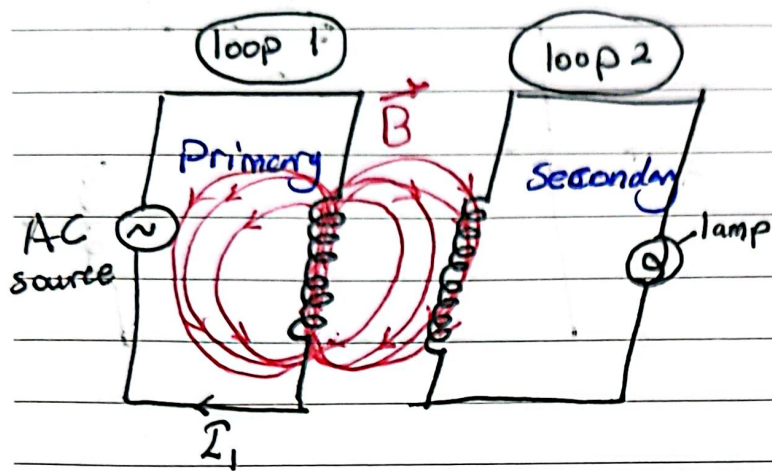
$$i = \frac{B}{\mu_0 n} \Rightarrow U_B = \frac{1}{2} L \left(\frac{B^2}{\mu_0^2 n^2}\right) = \frac{1}{2} (\mu_0 n^2 V) \left(\frac{B^2}{\mu_0^2 n^2}\right)$$

$$= \frac{1}{2} V \cdot \frac{B^2}{\mu_0}$$

$$\frac{U_B}{V} = \frac{1}{2} \frac{B^2}{\mu_0} \Rightarrow U_B = \frac{B^2}{2\mu_0}$$



Mutual Inductance



$$N_2 \Phi_{B_{21}} \propto I_1$$

↓
Magnetic flux through each turn of the secondary coil due to the current in the primary coil

Mutual inductance is a constant that only depends on the geometry and orientation of both coils. Its SI unit is Henry (H)

$$N_2 \Phi_{B_{21}} = M_{21} I_1$$

↓
Mutual inductance of secondary coil due to current in primary coil

$$\boxed{E_2 = -M_{21} \cdot \frac{dI_1}{dt}}$$

Note: in Vacuum, mutual inductance of both coils are equal.

$$\boxed{M_{12} = \frac{N_2 \Phi_{12}}{i_1}}$$

What is the difference between Self-Inductance (L) and Mutual Inductance (M)?

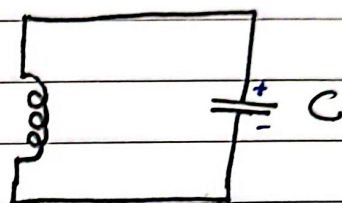
Self Inductance is about a single coil ~~of~~ interacting with itself.

Self induced voltage can be expressed as $E = -L \cdot \frac{di}{dt}$.

Mutual inductance involves two separate coils interacting with each other, through a shared magnetic field.

Oscillations in an LC circuit

if Capacitor is charged, it will discharge, where ~~the~~ inductor (L) will start storing energy in its magnetic fields, and once the capacitor has discharged its ~~pe~~ electrical potential energy, the inductor gives away the stored energy to charge the capacitor, and the cycle continues.



$$\Delta U_E + \Delta U_B = 0 \Rightarrow \frac{d^2 q}{dt^2} = -\frac{1}{LC} q$$

$$q = Q_{\max} \cos(\omega t + \phi)$$

$$\boxed{\omega = \frac{1}{\sqrt{LC}}}$$

similar to

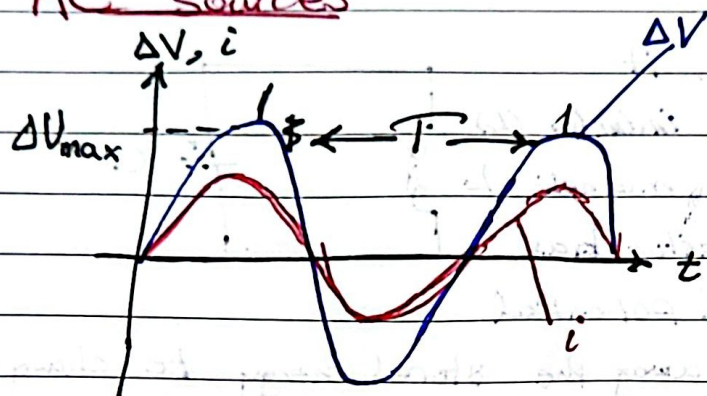
$$\frac{d^2 x}{dt^2} = -\frac{k}{m} x$$

$$\boxed{I_{\max} = \omega Q_{\max}}$$

Chapter 23: Alternating-Current Circuits

22/4/2026

AC sources



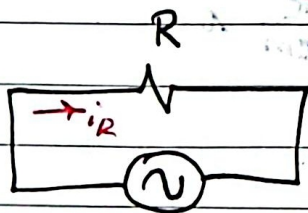
$$\Delta V = V_{\max} \sin \omega t$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

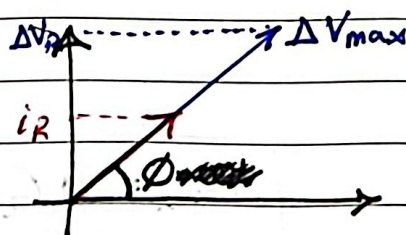
$$i = \frac{\Delta V}{R} \Rightarrow \Delta V = I_{\max} R \sin \omega t$$

note that the voltage and current are in phase, where they reach the max and min at the same time.

Phasor Diagram



$$\Delta V = \Delta V_{\max} \sin(\omega t)$$

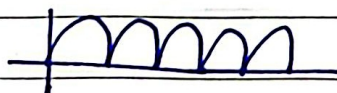


RMS current and Voltage

$$I_{\text{rms}} = \sqrt{(i^2)_{\text{avg}}}$$

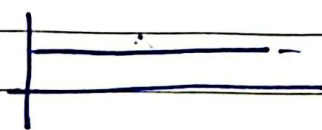
rms: root-mean-squared
so first square the wave

$$I_{\text{rms}} = \frac{I_{\max}}{\sqrt{2}} = 0.707 I_{\max}$$



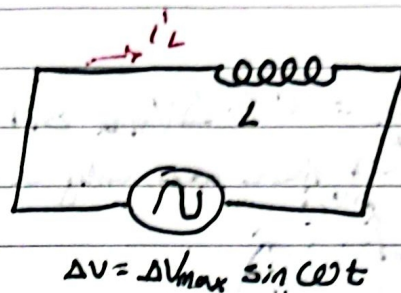
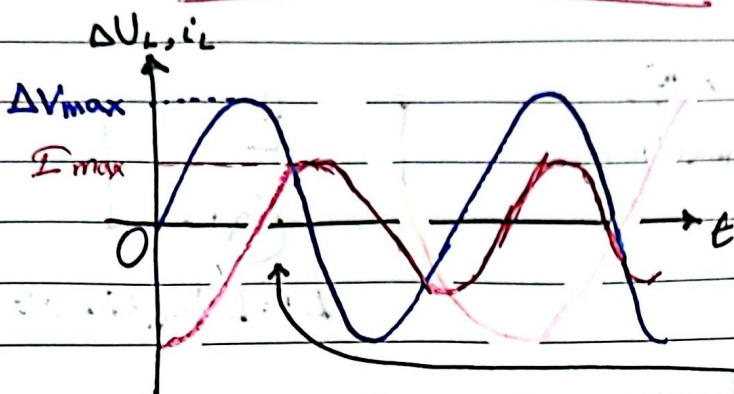
then take the mean

$$\Delta V_{\text{rms}} = \frac{\Delta V_{\max}}{\sqrt{2}} = 0.707 \Delta V_{\max}$$



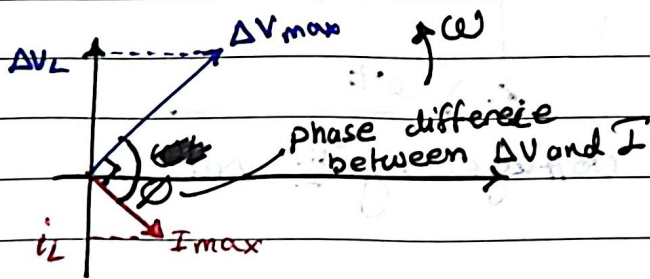
then take its square root

Inductors in an AC circuit



the current lags the voltage by 90° degrees ($\frac{\pi}{2}$ rad)

the voltage at $t=0$ reaches $\Delta V=0$, while the current is at $-I_{max}$, so the current takes exactly $\frac{\pi}{2}$ cycle to catch up and also reach zero, so current is delayed when compared to the voltage



$$X_L = \omega L \quad \leftarrow \text{magnitude of the impedance of the inductor}$$

$$I_{max} = \frac{V_{max}}{X_{max}}$$

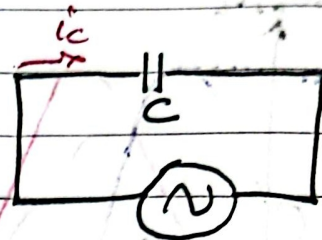
note, the impedance of inductors do NOT result in heat dissipation, but instead it just results in current being delayed

at some time $t = a$

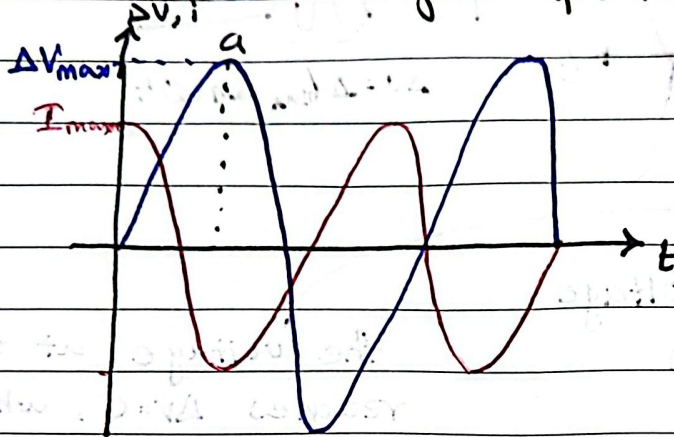
Capacitors in an AC circuit

$$i_c = \omega C \Delta V_{\max} \sin(\omega t + \frac{\pi}{2})$$

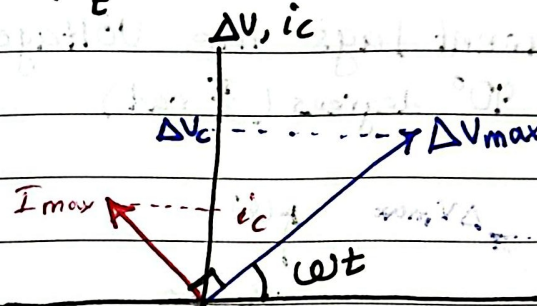
if there was only a capacitor



$$\Delta V = \Delta V_{\max} \sin(\omega t)$$



time domain



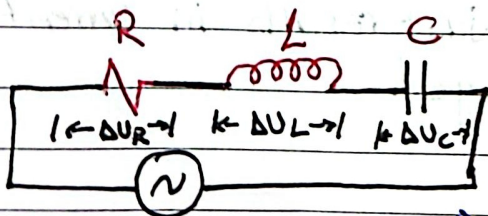
the current leads the voltage by $\frac{\pi}{2}$ rad.

Capacitive reactance

$$X_c = \frac{1}{\omega C}$$

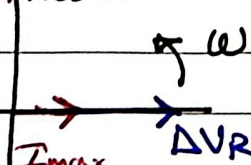
$$I_{\max} = \frac{\Delta V_{\max}}{X_c}$$

RLC series Circuit



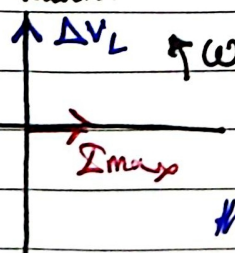
$$\Delta V_R = I_{\max} R \sin(\omega t)$$

Resistor

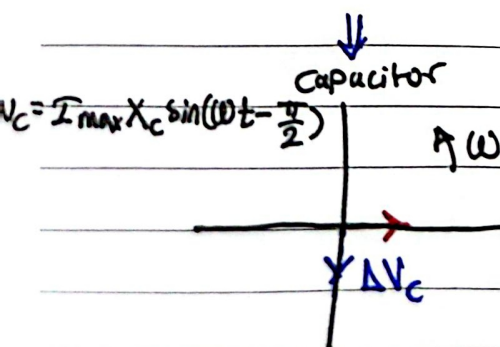


$$\Delta V_L = I_{\max} X_L \sin(\omega t + \frac{\pi}{2})$$

Inductor



note that for all components the have the same current through them, but they had different voltages across them at different times.



Impedance

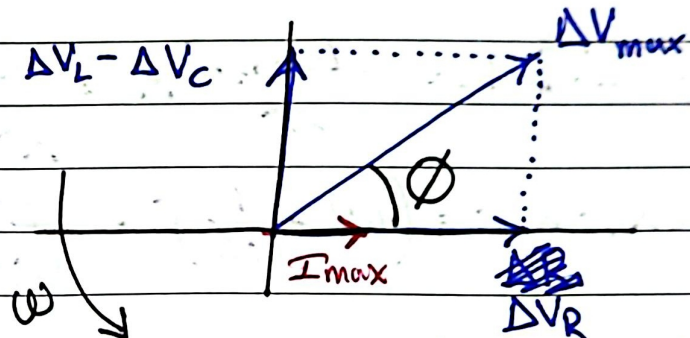
$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$I_{max} = \frac{\Delta V_{max}}{Z}$$

X_L and X_C are both on the imaginary dimension, so they are both added together, but since their phasors point in opposite directions, one would be positive and the other negative (specifically X_L is in $+j$ direction and X_C is in $-j$ direction)

$$\phi = \tan^{-1} \left(\frac{X_L - X_C}{R} \right)$$

Phase angle



Note: in an RLC circuit, the voltage phasor for each component in the circuit has a different angle with current. When V phasors are summed we get the total voltage phasor. The angle between this phasor and current tells us whether the circuit is mostly capacitive or inductive.

Power in an AC circuit

ϕ is the angle from voltage to the current

for a resistor whether the circuit is mostly capacitive or inductive

$$P_{avg} = I_{rms} \Delta V_{rms} \cos \phi$$

$$P_{avg} = I_{rms}^2 R$$

$$P_{avg} = \frac{1}{2} I_{max} \Delta V_{max} \cos \phi$$

$$U_B = \frac{1}{2} L I_{max}^2$$

ϕ is also the angle of the complex impedance vector derived from $\cos \phi = \frac{R}{Z}$

the maximum potential energy stored within a magnetic field of an inductor.

however this equation doesn't tell you whether voltage is leading or lagging the current. It only tells the magnitude of the angle between both.

to determine angle including the direction: $\tan(\phi) = \frac{X_L - X_C}{R}$

Resonance in a series RLC circuit

Resonance in a series RLC circuit occurs at a specific angular frequency ω_0 exactly ~~where~~ when the capacitive reactance equals the inductive reactance.

$$X_L = X_C$$

$$\omega_0 L = \frac{1}{\omega_0 C} \Rightarrow$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

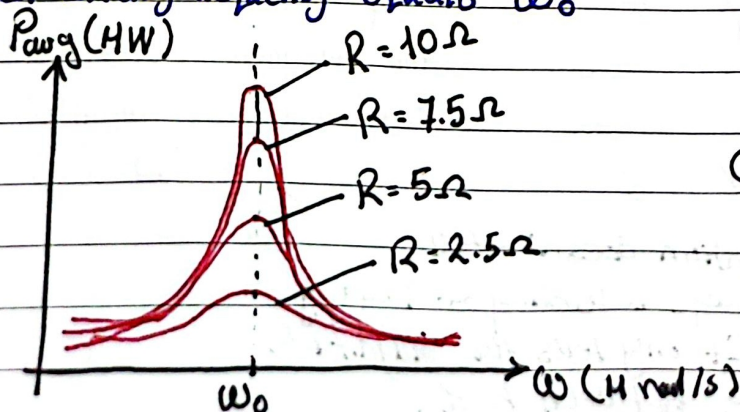
← the resonance angular frequency.

$$P_{avg} = I_{rms}^2 R = \frac{(\Delta V_{rms})^2}{Z^2} R = \frac{(\Delta V_{rms})^2 R}{R^2 + (X_L - X_C)^2}$$

$$(X_L - X_C)^2 = \left(\omega L - \frac{1}{\omega C} \right)^2 = \frac{L^2}{\omega^2} (\omega^2 - \omega_0^2)^2$$

$$P_{avg} = \frac{(\Delta V_{rms})^2 \cdot R \omega^2}{R^2 \omega^2 + L^2 (\omega^2 - \omega_0^2)^2}$$

the equation quantifies the average power dissipated by a series RLC circuit as a ~~function~~ continuous function of the driving angular frequency ω . It mathematically defines the resonance curves illustrated in the adjacent graph, demonstrating that maximum power transfer occurs precisely when driving frequency equals ω_0 .



$$Q = \frac{\omega_0}{\Delta\omega}$$

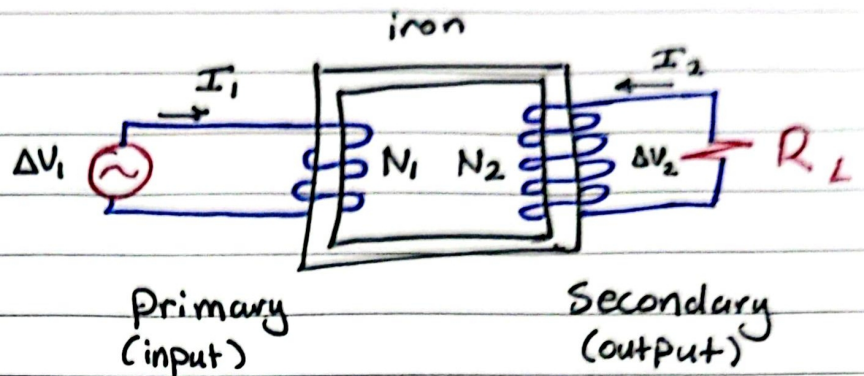
$$Q = \frac{\omega_0 L}{R}$$

The Transformer

$$\Delta V_1 = -N_1 \cdot \frac{d\Phi_B}{dt}$$

$$\Delta V_2 = -N_2 \cdot \frac{d\Phi_B}{dt}$$

$$\Delta V_2 = \frac{N_2}{N_1} \Delta V_1$$



When the source applies a voltage ΔV_1 to the primary coils, the power demanded by the secondary load (R_L) forces the primary circuit to draw a specific current, I_1 .

In the primary coil, while the ohmic resistance is zero, there is induced back-emf that isn't small due to the solenoid, and it is generating an imaginary resistance which is the inductive reactance, ~~xxxx~~ making Z non-zero. ($X_L = \omega L$)

$I_P = \frac{V_P}{Z}$. So, the current in the primary coil is not infinite or something like that.

$$R_{eq} = \left(\frac{N_1}{N_2} \right)^2 R_L$$